# COMPUTER - AIDED SYNTHESIS AND ANIMATION OF PLANAR FOUR - LINK MECHANISMS
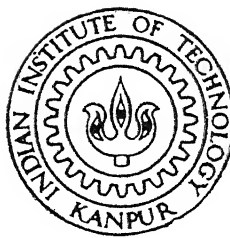
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NITEESH KUMAR DUBEY
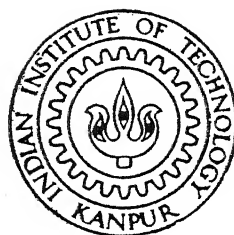
DEPARTMENT OF MECHANICAL ENGINEERING

## INDIAN INSTITUTE OF TECHNOLOGY KANPUR

FEBRUARY, 1998

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# COMPUTER-AIDED SYNTHESIS AND ANIMATION OF PLANAR FOUR-LINK MECHANISMS

*A Thesis Submitted*

in Partial Fulfillment of the Requirements

for the Degree of

## Master of Technology

*by*

## NITEESH KUMAR DUBEY

*to the*

**DEPARTMENT OF MECHANICAL ENGINEERING**

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

February 1998

ME-1998-M-DUB-COM

Entered in the Centum

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# CERTIFICATE

This is to certify that the thesis , entitled , "COMPUTER-AIDED

SYNTHESIS AND ANIMATION OF PLANAR FOUR-LINK MECHANISMS " by

**Niteesh Kumar Dubey** is a record of work carried out under my supervision and has not

been submitted elsewhere for a degree .


**Dr. A. K. MALLIK**
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Feb , 1998

Dedicated to

my

beloved and holy mother.

# ACKNOWLEDGEMENT

# ABSTRACT

Like most design problems , kinematic synthesis of planar linkages involves a number of trials to reach an acceptable or an optimal solution . Both graphical and analytical methods are available for the synthesis of four-link planar mechanisms . Independent of the method used , a solution arrived at always needs to be verified through animation . Consequently , a versatile software incorporating graphical and analytical methods and capable of analysis , synthesis and animation will be very useful to a mechanism designer .

This thesis , as a part of a D. S. T. ( Department of Science and Technology ) sponsored project , attempts to develop some specific problems , the solution of which require special graphical techniques or overall optimization problem . The Borland $C^{++}$ language with graphic facility has been used because of its certain special features , which are eminently suitable for the problems under consideration . The software informs the user the kind of problems it can handle , asks for the input data ( including designer ' s choice ) , verifies the feasibility of the existence of solution , obtains the optimal solution and demonstrates the performance of the obtained solution .

# CONTENTS

# NOMENCLATURE

| | | |
|---|---|---|
| **H** | = | Stroke-length of the slider of a slider-crank mechanism |
| $Q_{RR}$ | = | Quick-return ratio of a slider-crank mechanism |
| $\theta_i$ | = | Angle of rotation of the $i^{th}$ link of a four-link mechanism |
| $\theta_2{}^*$ | = | Swing angle of the crank ( CCW ) when the rocker ( or slider of a slider-crank mechanism ) goes from outer dead-center to inner dead-center . |
| $\theta_4{}^*$ | = | Rocker swing angle corresponding to $\theta_2{}^*$ |
| $\theta_{24}{}^*$ | = | ( $\theta_2{}^* - \theta_4{}^*$ ) |
| **E** | = | The offset a slider-crank mechanism |
| $L_i$ | = | Link length of the $i^{th}$ link of a four-link mechanism |
| $R , \gamma$ | = | The parameters , used to define a coupler point on the coupler of ( or the connecting rod of a 3R-1P mechanism ) a 4R mechanism |
| $\mu$ | = | Transmission angle of a four-link mechanism |
| $\theta_{2c}$ | = | Swing angle of the crank ( CCW ) when a coupler point of a crank-rocker mechanism goes from one cusp to another . |
| $\theta_{4c}$ | = | Swing angle of the rocker ( CCW ) corresponding to $\theta_{2c}$ |
| $\omega_i$ | = | Angular velocity of the $i^{th}$ link of a four-link mechanism |
| $\alpha_i$ | = | Angular acceleration of the $i^{th}$ link of a four-link mechanism |

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# 1. INTRODUCTION

## 1.1 INTRODUCTION

Planar linkages are extensively used in various machineries in the field of agriculture, printing, textile and many others. A successful design of such a machinery very often requires, as a first step, an optimal kinematic design of various planar linkages used in it.

The kinematic design of a linkage starts with the number synthesis, i.e., with the decision on the number of links and kinematic pairs to be used for attaining the desired objective. However, a well-developed theory exists only for designing four-link mechanisms. The theory also points out, in some cases, the limitations of such four-link mechanisms. Only when the job in hand can not be achieved by a four-link mechanism, one needs to go for more complicated ( e.g., six-link or eight-link ) mechanisms. Even for designing such higher order linkages, very often the theory of four-link mechanisms serves as the starting point.

Over the last hundred years, various graphical and analytical techniques have been developed for the dimensional synthesis ( i.e., the determination of the actual kinematic dimensions ) of four-link planar mechanisms. The suitable choice of a method depends on the type of problem. In a four-link mechanism, the links connected to the fixed link ( ground or frame of reference ) are referred to as the input and output links, whereas the link not directly connected to the fixed link is called the floating link ( e.g., the coupler, the connecting rod ).

Depending on the required kinematic characteristics to be satisfied by the designed linkage, dimensional synthesis problems can be broadly classified as

[i] **Motion Generation** - It is defined as the control of a *line segment* in a plane ( on the floating link ) such that it assumes some prescribed set of sequential positions. The guidance may or may not be coordinated with the input motion.

**[ii] Path Generation** - If a tracer point on the floating link has to traverse through some specified points ( on a specified path ), then such a problem is classified as a problem of path generation. The generation of a prescribed path may or may not be coordinated with the input motion. No attempt is made to control the orientation of the link which contains the tracer point.

**[iii] Function Generation** - In this problem, the movements of the output and input links are coordinated according to a prescribed function [ say, y = f (x) ]. The change in the input movement corresponds to the change in the independent variable and that in the output movement to the change in the dependent variable. A function generator is conceptually a " black box " which delivers some predictable output in response to a known input.

**[iv] Dead-Center Problems** - These types of problems generally occur where the quick-return action of the output link for a constant input speed becomes important. Here, the linkage dimensions are determined for fulfilling the prescribed dead-center configurations.

In a real-life problem of path generation or function generation, it is generally not possible to design a mechanism which achieve the specified characteristics over the desired range of movements. One can only attempt for an approximate solution. There are two approaches to achieve an acceptable level of approximation.

**[a] Accuracy Point Approach** - In this approach, the generated characteristics of the designed mechanism exactly match with the desired characteristics only at some discrete positions within the specified range of movement. But at remaining configurations there exists some difference between the desired and generated characteristics which is called the *structural error*. One should select those precision points within the specified range of movement for which the maximum structural error becomes as small as possible. As a first approximation, *Chebyshev's accuracy points* [1] are generally chosen. With this approach, we can get a better solution if we take more number of precision points within the specified range of movement, but the number of precision points is limited by the number of available design parameters.

[b] **Optimization Approach** - In contrast to the above approach, here, the structural error, which may not be zero anywhere, is minimized in an overall sense in the entire range. It should be noted that, one can choose as many design positions as he / she wants. The number of design positions is independent of the number of available design parameters.

The accuracy point approach can be adopted while applying either a graphical or an analytical method, but the optimization approach is amenable to only analytical methods.

Finally, independent of the approach or technique adopted, a designed linkage has always to be checked for branch and order defects by checking through the entire range of movement. Moreover, there are always some assumed parameters and an optimal solution can be attained only after a number of trials.

All the points mentioned above make it amply clear that the dimensional synthesis of four-link mechanisms is an ideal problem to be tackled by a computer software with animation and graphic display. Commercial softwares are, of course, available, which are mostly suitable for kinematic and dynamic analysis. In this thesis, an attempt has been made, as a part of a D.S.T. ( Department of Science and Technology ) sponsored project, to develop a software which encompasses certain special graphical methods and optimisation approaches to solve a variety of dimensional synthesis problems of four-link planar mechanisms.

## 1. 2 OBJECTIVE AND SCOPE OF THE PRESENT WORK

The objective of this work is to develop a computer software which can synthesize and animate four-link mechanisms for different types of problems, which are

- *Dead-center problems for crank-slider and crank-rocker mechanisms* : Here

  [i] the quick-return ratio,

  [ii] the stroke-length ( in the case of 3R-1P ) or the rocker swing angle ( in the case of crank-rocker ) and

  [iii] one of the link lengths or offset

have to be given as inputs. This kind of problem has to be solved while designing quick-return mechanisms.

- *Generation of coupler curves* :  To draw a segmented coupler curve corresponding to a specified coupler point on the floating link. Each segment corresponds to a specified crank-rotation. Thus the velocity of the coupler point at different positions can be judged by observing the length of each segment. Larger the length, greater is the velocity of the coupler point at that position.

- *Path generation by optimization approach* :  In this case, the coordinates of several points on the specified path, coordinated with input angles will be required as inputs. In some cases, where the shape of the path along with the structural errors at some specified points are important, this approach is usually necessary.

- *Function generation by optimization approach* :  Here, either a function or a set of coordinated input-output angles are prescribed. This is generally used in the synthesis of function generators.

- *Two cusps problems* :  Here, a crank-rocker has to be designed so that a coupler point curve has two cusps at prescribed locations. Further, the crank rotation as the coupler point moves from one cusp to another is also prescribed. This type of problem arises in several processes ( e.g. in the stamping and feeding processes, where a coupler point has to reach at its destination and return back without impact and shock, noiseless rocker-stop used in typewriters, recording machine in which a recording pencil writes only between two limiting points etc. ).

- *To find the coupler curve cognates* :  In the case of a path generation problem, a designed mechanism may not be accepted if its fixed pivots are located outside the available space. Then, in such a situation, its cognate generating the same coupler curve may be accepted. Similarly, if the initially designed mechanism turns out to be a Grashofian double-rocker, then its crank-rocker cognate may be an acceptable design.

- *Problems with specified kinematic variables ( angular velocities and accelerations )* : To design a 4R linkage so that at a particular configuration, the prescribed values of

[i] angular displacements, velocities and accelerations of the input and output links or

[ii] angular velocities and accelerations of the input, floating and output links are achieved.

To meet the computational and animation requirements for tackling the above classes of problems, we have selected $C^{++}$ ( Borland, with graphics environment ) as the programming language because of its special characteristics ( e.g. classes, objects, inheritance, overloading operator, data hiding etc. ). The classes and some special member functions used in the present work are listed in the appendix.

# 2. DEAD - CENTER PROBLEMS

## 2. 1 SLIDER - CRANK MECHANISM

Let us first consider the problem of synthesizing a slider-crank mechanism with prescribed stroke-length and quick-return ratio.

Referring to Fig 2.1, suppose

$B_1$ and $B_2$ represent the extreme positions of the slider 4 ( when the crank 2

and the connecting rod 3 become collinear ),

H = stroke-length of the slider = $B_1B_2$,

$Q_{RR}$ = quick-return ratio,

$\theta_2*$ = An angle through which the crank rotates when

the slider goes from $B_1$ to $B_2$

or $\quad \theta_2* = 2\pi \; \{ \; ( Q_{RR} ) / ( 1 + Q_{RR}) \; \}$,

$\theta_2** = \angle B_1O_2B_2 = ( \theta_2* - \pi \; )$

and $\quad \beta = \angle O_2B_1B_2$.

If only H and $Q_{RR}$ ( or $\theta_2**$ ) are given, then we can have infinite solutions. It can be understood easily by using the property of a circle which states that " a chord of a circle subtends equal angle at all the circumferencial points which are on the same side of the chord ". So $O_2$ can be placed anywhere on the circumference of a circle ( $CR_2$ ) passing through $B_1$, $B_2$ and $O_2$. Thus, for a unique design, besides H and $\theta_2*$ ( i.e., the quick-return ratio ), one more kinematic dimension, viz., $L_2$ ( crank length ) or $L_3$ ( connecting rod length ) or E ( offset ), must be either prescribed or assumed. Depending on which one of the three dimensions is given ( or assumed ), three methods of determining the other two are discussed below.

**Case 1.** H, $\theta_2*$ and E are prescribed, $L_2$ and $L_3$ are to be determined :

From $\Delta$ 's $O_2NB_1$ and $O_2NB_2$,

$$E = ( L_3 - L_2 ) \sin ( \beta + \theta_2** ) = ( L_3 + L_2 ) \sin ( \beta ) \qquad [\,2.1.1\,]$$

and

$$H = ( L_3 + L_2 ) \cos ( \beta ) - ( L_3 - L_2 ) \cos ( \beta + \theta_2** ). \qquad [\,2.1.2\,]$$

From Equations 2.1.1 and 2.1.2,

$$\sin ( \beta + \theta_2** ) \sin ( \beta ) = ( E / H ) \sin ( \beta + \theta_2** - \beta )$$

or

$$\cos ( 2\beta + \theta_2** ) = \cos ( \theta_2** ) - ( 2E / H ) \sin ( \beta + \theta_2** - \beta )$$

or

$$\beta = [\, \cos^{-1}\{ \cos ( \theta_2** ) - ( 2E / H ) \sin ( \theta_2**) \} - \theta_2** \,] / 2.$$

$$[\,2.1.3\,]$$

Now

$$O_2B_1 = ( L_3 + L_2 ) = E / \sin ( \beta ) \qquad [\,2.1.4\,]$$

and

$$O_2B_2 = ( L_3 - L_2 ) = E / \sin ( \beta + \theta_2** ). \qquad [\,2.1.5\,]$$

Therefore,

$$L_3 = ( O_2B_1 + O_2B_2 ) / 2 \quad \text{and} \quad L_2 = ( O_2B_1 - O_2B_2 ) / 2. \qquad [\,2.1.6\,]$$


**Case 2.** H, $\theta_2*$ and L$_3$ are prescribed, L$_2$ and E are to be determined :


From $\Delta$ O$_2$B$_2$B$_1$,

$$\cos ( \theta_2** ) = \{ ( L_3 + L_2 )^2 + ( L_3 - L_2 )^2 - H^2 \} / \{ 2 ( L_3 + L_2 ) ( L_3 - L_2 ) \}$$

or

$$\cos ( \theta_2** ) = \{ L_3^2 + L_2^2 - ( H^2 / 2 ) \} / ( L_3^2 - L_2^2 ), \qquad [\,2.1.7\,]$$

which gives

$$L_2 = \sqrt{ [\, [ ( H^2 / 2 ) - L_3^2 \{ 1 - \cos ( \theta_2** ) \} ] / \{ 1 + \cos ( \theta_2** ) \} \,] }.$$

$$[\,2.1.8\,]$$

From Equation 2.1.1, we get

$$( L_3 - L_2 ) \{ \sin ( \beta ) \cos ( \theta_2^{**} ) + \cos ( \beta ) \sin ( \theta_2^{**} ) \}$$

$$= ( L_3 + L_2 ) \sin ( \beta )$$

or

$$\tan ( \beta ) = \{ ( L_3 - L_2 ) \sin ( \theta_2^{**} ) \} / \{ ( L_3 + L_2 ) - ( L_3 - L_2 ) \cos ( \theta_2^{**} ) \}$$

or

$$\beta = \tan^{-1} [ \{ ( L_3 - L_2 ) \sin ( \theta_2^{**} ) \} / \{ ( L_3 + L_2 ) - ( L_3 - L_2 ) \cos ( \theta_2^{**} ) \} ].$$

$$[ 2.1.9 ]$$

Hence

$$E = ( L_3 + L_2 ) \sin ( \beta ). \qquad\qquad [ 2.1.10 ]$$

**Case 3.** H, $\theta_2^*$ and $L_2$ are prescribed, $L_3$ and E are to be determined :

From Equation 2.1.7,

$$L_3 = \sqrt{ [ \ [ ( H^2 / 2 ) - L_2^2 \{ 1 + \cos ( \theta_2^{**} ) \} ] / \{ 1 - \cos ( \theta_2^{**} ) \} \ ] }.$$

$$[ 2.1.11 ]$$

Then using Equations 2.1.9 and 2.1.10 we can get the offset E.

**Optimal solution :** As we have seen above, that besides H and $Q_{RR}$ ( or $\theta_2^*$ ), one has to give either $L_2$ or $L_3$ or E to obtain a unique solution for a 3R-1P mechanism. If only H and $Q_{RR}$ ( or $\theta_2^*$ ) are available then one can also design a unique slider-crank mechanism for which the minimum transmission angle is maximum.

For smooth functioning of a mechanism in real life, we should ensure that, besides satisfying the kinematic requirements, the mechanism moves freely. Neglecting the effects of the gravity, inertia and friction all binary links become two force members. Consequently, the force in any such link is along the link vector and the force analysis becomes simple. To judge the free running quality of a mechanism, the transmission angle is considered as one of the most important parameter.

For a slider-crank mechanism the *transmission angle* is defined as the absolute acute angle between the connecting rod and the normal to the slider movement ( Fig - 2.2 ). The minimum transmission angle [1] for this mechanism is given by

$$\mu_{min} = \cos^{-1} [ ( L_2 + E ) / L_3 ]. \qquad\qquad [ 2.1.12 ]$$

Referring to Fig 2.1,

from $\Delta\ O_2B_1B_2$, we observe that

$$L_3 - L_2 = \{ H / \sin ( \theta_2{}^{**} ) \} \sin ( \beta )$$

and

$$L_3 + L_2 = \{ H / \sin ( \theta_2{}^{**} ) \} \sin ( \beta + \theta_2{}^{**} ).$$

Therefore,

$$L_2 = \left[ \frac{H}{2 \sin ( \theta_2{}^{**} )} \right] \{ \sin ( \beta + \theta_2{}^{**} ) - \sin ( \beta ) \} \qquad [ 2.1.13 ]$$

and

$$L_3 = \left[ \frac{H}{2 \sin ( \theta_2{}^{**} )} \right] \{ \sin ( \beta + \theta_2{}^{**} ) + \sin ( \beta ) \}. \qquad [ 2.1.14 ]$$

Again, from $\Delta\ O_2B_1N$, we get

$$E = ( L_3 + L_2 ) \sin ( \beta )$$

or

$$E = \left[ \frac{H}{\sin ( \theta_2{}^{**} )} \right] \sin ( \beta + \theta_2{}^{**} ) \sin ( \beta ). \qquad [ 2.1.15 ]$$

To maximize the minimum transmission angle $\mu_{min}$ ( given by Equation 2.1.12 ), one must minimize the value of $[ ( L_2 + E ) / L_3 ]$ which is a function of the variable $\beta$. Using Equations 2.1.13, 2.1.14 and 2.1.15, we get

$$\frac{L_2 + E}{L_3} = 1 - \left[ \frac{2 \sin ( \beta ) \{ 1 - \sin ( \beta + \theta_2{}^{**} ) \}}{\sin ( \beta + \theta_2{}^{**} ) + \sin ( \beta )} \right]. \qquad [ 2.1.16 ]$$

From Fig 2.1, it is obvious that $O_2$ can not be above the line passing through $B_1$ and $B_2$ ; and $O_2B_1$ must not be less than $O_2B_2$ ( since $B_1$ and $B_2$ are the outer and inner dead-centers, respectively ). This implies that $\beta$ varies from 0 to ( $\pi - \theta_2^{**}$ ) / 2.

Thus, we have a function [ ( $L_2 + E$ ) / $L_3$ ] of a single variable $\beta$. The variable $\beta$ varies from 0 to ( $\pi - \theta_2^{**}$ ) / 2 . As we know that, to minimize a single variable function for a given range of the variable, one can use the *golden section search method* [2]. For the present problem, using this optimization method, we get $\beta_{opt}$ corresponding to the minimum value of [ ( $L_2 + E$ ) / $L_3$ ] and hence $L_2$, $L_3$, E and $\mu_{min}$ ( from Equations 2.1.13, 2.1.14, 2.1.15 and 2.1.12 respectively ) are known.

**Flow chart with illustrations -** Using the computational flow chart ( Chart 2.1 ), one can easily understand the steps used in the program for dead-center problems of a slider-crank mechanism. It also helps the designer to run the program. After computing the design parameters of the mechanism, the software itself checks the existence of the mechanism. If the mechanism exists, then DRAW_3R1P( ) can be used to animate the slider-crank mechanism. For verification of the results, one should opt for *dead-center configurations* which display all the dead-center configurations along with the stroke-length.

To illustrate the synthesis of a slider-crank mechanism ( case **1** ), we have considered a problem with    H = 6.0 unit, $Q_{RR} = 1.4$ and E = 1.0 unit.
For this problem, the final design yields

$$L_2 = 2.8629 \text{ unit,}$$

$$L_3 = 4.489 \text{ unit}$$

and    $\mu_{min} = 30.73^0$.

To verify the result, one can see its dead-center configurations shown in Plot 2.1.

For an optimal design problem, taking H = 6.0 unit and $Q_{RR} = 1.4$, we design a slider-crank mechanism for which

$$L_2 = 2.7307 \text{ unit,}$$

$$L_3 = 5.5222 \text{ unit,}$$

$$E = 1.9198 \text{ unit}$$

and $\quad \mu_{min} = 32.63^0.$

Its dead-center configurations are shown in Plot 2.2.

## 2. 2 CRANK - ROCKER MECHANISM

Now let us consider the synthesis of a crank-rocker mechanism for dead-center problems. From Fig 2.3, $O_2A_1B_1O_4$ ( shown in Fig 2.3 ) is a crank-rocker mechanism. The rocker $O_4B_1$ swings by an angle $\theta_4*$ when the crank $O_2A_1$ rotates by $\theta_2*$ ( CCW ) and correspondingly the moving hinge of the follower traverses from $B_1$ ( outer-dead center ) to $B_2$ ( inner-dead center ). The link lengths ( $L_1$, $L_2$, $L_3$ and $L_4$ ) are indicated in the figure.

Here $B_1B_2 = 2 L_4 \sin ( \theta_4* / 2 )$ and the offset E can be taken as the perpendicular distance of line $B_1B_2$ from $O_2$. So if the length $B_1B_2$ ( $\equiv$ the stroke-length ), $Q_{RR}$ ( or $\theta_2*$ ) and one of the lengths ( $L_1$ or $L_2$ or $L_3$ or E ) are given, then one can synthesize a crank-rocker mechanism in the same way as was done for the slider-crank mechanism.

But in real life, generally $Q_{RR}$ ( or $\theta_2*$ ), the rocker swing angle $\theta_4*$ and the fixed length $L_1$ are given to synthesize a crank-rocker mechanism. In such a situation, the solution is obtained by the Alt's construction [1]. The Alt's construction gives infinite solutions, but one must select the mechanism for which the minimum transmission angle is maximum considering the free running quality of the mechanism.

The *transmission angle* of a 4R mechanism is defined as the absolute value of the acute angle between the output link and the coupler ( Fig 2.4 ). Suppose a torque $T_2$ is applied at link 2, then the generated torque $T_4$ (on link 4 ) will be directly proportional to the force $F_t [ = F \sin( \mu ) ]$ where $\mu$ is the transmission angle and F is the produced axial force in the coupler. The radial component $F_r$ provides only tension or compression in link 4. For good performance of a mechanism it is suggested that $\mu_{min} > 30^0 - 35^0.$

The transmission angle [1] $\mu$ ( for the crank-rocker ) is given by

$$\mu = \cos^{-1} [ \{ ( L_3^2 + L_4^2 ) - ( L_1^2 + L_2^2 ) - 2 L_1 L_2 \cos \theta_2 \} / ( 2 L_3 L_4 ) ]. \quad [ 2.2.1 ]$$

If $\theta_2{}^* \geq \pi$ [ or $( L_3{}^2 + L_4{}^2 ) \geq ( L_1{}^2 + L_2{}^2 )$ ] then

$$\mu_{min} = \cos^{-1} [ \{ ( L_3{}^2 + L_4{}^2 ) - ( L_1{}^2 + L_2{}^2 ) + 2 L_1 L_2 \} / ( 2 L_3 L_4 ) ]$$

[ 2.2.2 ]

and if $\theta_2{}^* \leq \pi$ [ or $( L_3{}^2 + L_4{}^2 ) \leq ( L_1{}^2 + L_2{}^2 )$ ] then

$$\mu_{min} = \pi - \cos^{-1} [ \{ ( L_3{}^2 + L_4{}^2 ) - ( L_1{}^2 + L_2{}^2 ) - 2 L_1 L_2 \} / ( 2 L_3 L_4 ) ].$$

[ 2.2.3 ]

To get the optimum solution, the designer generally uses Volmer's nomogram ( Plot 2.4 ). To design a crank-rocker mechanism for prescribed dead-center configurations on a computer, one must have to feed the data from Volmer's nomogram. But here, we have derived a single variable function ( say $\mu_{min}$ ) which is again optimized by the *golden section search method* to obtain the corresponding unknown design parameters ( $L_2$, $L_3$ and $L_3$ ). Finally we verified the results with the Volmer's nomogram.

Using the Alt's constructions, the unknown design parameters of a crank-rocker mechanism for various situations are presented below. In all the cases $\theta_2{}^*$ and $\theta_4{}^*$ are measured CCW from the outer to the inner dead-center configurations.

Let

$$\theta_2{}^{**} = \theta_2{}^* - \pi \quad \text{and} \quad \theta_{24}{}^* = \theta_2{}^* - \theta_4{}^*.$$

[ 2.2.4 ]

**Case 1 :** [ $\theta_{24}{}^* < \pi$ and $\theta_2{}^* \geq \pi$ ]

Referring to Fig 2.5, $O_2$ and $O_4$ are the fixed pivots such that $O_2 O_4 = L_1$. At $O_2$, a line $O_2 E_2$ is drawn at an angle of ( $- \theta_2{}^* / 2$ ) to $O_2 O_4$, and another line $O_4 E_4$ is drawn through $O_4$ at an angle of ( $- \theta_4{}^* / 2$ ) to $O_2 O_4$. These two lines intersects at F and the circle $CR_1$ is drawn with $O_2 F$ as the diameter. The mid-normal of $O_2 F$ intersects $O_4 F$ at $C_2$. Another circle $CR_2$, is drawn with its center at $C_2$ and radius as $C_2 F$.

In this case, since $C_2F$ is greater than ( if $\theta_2^* > \pi$ ) or equal to ( if $\theta_2^* = \pi$ ) $C_2O_4$. The circle $CR_2$ intersects $O_2O_4$ at S which coincides with $O_4$ ( if $\theta_2^* = \pi$ ) or lies outside ( if $\theta_2^* > \pi$ ) $O_2O_4$ ( as shown in Fig 2.5 ).

A line $O_4N$ is drawn through $O_4$ at an angle of ( $- \theta_4^*$ ) to $O_2O_4$ intersecting the circle $CR_2$ at the point N. The point $B_1$ can now be taken anywhere on the arc NS. The line $O_2B_1$ intersects $CR_1$ at $A_1$. Thus, $O_2A_1B_1O_4$ is the required 4R crank-rocker linkage at its outer dead-center configuration. All the unknowns and some of the assumed symbols are shown in the figure.

Now to determine the unknown design parameters, referring to Fig 2.5,

let

$$C_1 \equiv (x_{c1}, y_{c1}), \qquad r_1 = \text{the radius of circle } CR_1,$$

$$C_2 \equiv (x_{c2}, y_{c2}) \quad \text{and} \quad r_2 = \text{the radius of circle } CR_2.$$

From $\triangle\ O_2FO_4$

$$r_1 = O_2F\, /\, 2$$

or

$$r_1 = \{\ L_1 \sin (\ \theta_4^*/2\ )\ \}\, /\, \{\ 2 \sin (\ \theta_{24}^*/2\ )\ \} \qquad [\,2.2.5\,]$$

$$[\because \angle\ O_2FO_4 = \theta_{24}^* /\, 2\,],$$

$$x_{c1} = -\, r_1 \cos (\ \theta_2^* /\, 2\ ) \qquad\qquad [\,2.2.6\,]$$

and $\quad y_{c1} = r_1 \sin (\ \theta_2^* /\, 2\ ).$ $\qquad\qquad\qquad$ $[\,2.2.7\,]$

From $\triangle\ C_1FC_2$

$$r_2 = FC_2 = r_1 \sec (\ \theta_{24}^*/\, 2\ ), \qquad\qquad [\,2.2.8\,]$$

from $\triangle\ O_2FO_4$

$$x_{c2} = 2\, x_{c1} + r_2 \cos (\ \theta_4^* /\, 2\ ) \qquad\qquad [\,2.2.9\,]$$

$$[\because \angle O_2O_4F = (\ \theta_4^*\ )/\, 2\ ]$$

and

$$y_{c2} = 2\, y_{c1} - r_2 \sin (\ \theta_4^* /\, 2\ ). \qquad\qquad [\,2.2.10\,]$$

Let

$$N \equiv ( x_n, y_n ) \text{ and } O_4N = l_n,$$

where

$$x_n = L_1 - l_n \cos ( \theta_4^* ) \qquad\qquad [ 2.2.11 ]$$

and $\quad y_n = l_n \sin ( \theta_4^* ).$ $\qquad\qquad [ 2.2.12 ]$

Since N is on the circle $CR_2$, we can write

$$( x_n - x_{c2} )^2 + ( y_n - y_{c2} )^2 = r_2^2.$$

Putting the values of $x_n$ and $y_n$ in the above equation and on simplification, we get

$$l_n^2 + \{ 2 ( x_{c2} - L_1 ) \cos ( \theta_4^* ) - 2 y_{c2} \sin ( \theta_4^* ) \} l_n + ( L_1^2 - 2 L_1 x_{c2} ) = 0.$$

$$[ 2.2.13 ]$$

Equation 2.2.13 gives two values of $l_n$. Here, the positive value of $l_n$ is chosen as $\dot{y}_n$ is positive. Then, from Equations 2.2.11 and 2.2.12, we can have $x_n$ and $y_n$.

Now

$$\tan \beta_n = y_n / x_n$$

or $\quad \beta_n = \tan^{-1} ( y_n / x_n ) .$ $\qquad\qquad [ 2.2.14 ]$

For the point S, $\qquad \beta_s = 0.$

Thus, $\beta$ can vary from $\beta_s$ to $\beta_n$.

Now let us consider a general point $B_1$ on the circle $CR_2$ at an angle $\beta$ ( which may vary from $\beta_s$ to $\beta_n$ ).

Suppose $\qquad O_2B_1 = L = L_2 + L_3.$

Therefore,

$$( L \cos \beta - x_{c2} )^2 + ( L \sin \beta - y_{c2} )^2 = r_2^2$$

or

$$L = 2 ( x_{c2} \cos \beta + y_{c2} \sin \beta ) \quad [ \because x_{c2}^2 + y_{c2}^2 = r_2^2 ]. \quad [ 2.2.15 ]$$

Similarly the point $A_1$ lies on the circle $CR_1$ at the same angle $\beta$, therefore,

$$( L_2 \cos \beta - x_{c1} )^2 + ( L_2 \sin \beta - y_{c1} )^2 = r_1^2$$

or

$$L_2 = 2 ( x_{c1} \cos \beta + y_{c1} \sin \beta ) \qquad [ \because x_{c1}^2 + y_{c1}^2 = r_1^2 ]. \quad [ 2.2.16 ]$$

Now

$$L_3 = L - L_2 \qquad\qquad [ 2.2.17 ]$$

and from $\triangle\ O_2 B_1 O_4$

$$L_4 = ( L^2 + L_1^2 - 2 L_1 L \cos \beta )^{1/2}. \qquad [ 2.2.18 ]$$

Thus, for any angle $\beta$ (varying from $\beta_s$ to $\beta_n$) we can have the link lengths ($L_1$, $L_2$, $L_3$ and $L_4$) to design the crank-rocker mechanism. But we have already discussed that the mechanism for which $\mu_{min}$ is maximum will be the best regarding the transmi-ssion quality. Now we have to maximize the minimum transmission angle (given by Equation 2.2.2)

$$\mu_{min} = \cos^{-1} [ \{ ( L_3^2 + L_4^2 ) - ( L_1^2 + L_2^2 ) + 2 L_1 L_2 \} / ( 2 L_3 L_4 ) ].$$

$$[ 2.2.19 ]$$

Since $L_2$, $L_3$ and $L_4$ are functions of $\beta$, hence $\mu_{min}$ is also a function of the same variable $\beta$ (varying from $\beta_s$ to $\beta_n$). To maximize $\mu_{min}$, we have used the *golden section search method*. As we know that this optimization method minimizes a single variable function in a given range of its variable. Therefore, in the present problem, we have chosen ($-\mu_{min}$) (since $\mu_{min}$ is to be maximized) as a function and $\beta$ as a variable.

After optimization, we can get $\beta_{opt}$ (corresponding to ($\mu_{min}$)$_{max}$) and hence the desired unknown link lengths ($L_2$, $L_3$ and $L_4$).

**Case 2 : [ $\theta_{24}^* < \pi$ and $\theta_2^* < \pi$ ]**

Referring to Fig 2.6, in this case, the point $O_4$ lies outside the circle $CR_2$ because $FC_2 < C_2O_4$. Hence, the line $O_4N$ (at an angle $-\theta_4^*$) cuts the circle $CR_2$ at two points both with positive y coordinates.

From Equation 2.2.13, one value of $l_n$ corresponds to the point N and the other to

the point S. Therefore, using Equations 2.2.11, 2.2.12 and 2.2.14 we can have $\beta_s$ and $\beta_n$. Further, from Equation 2.2.3,

$$\mu_{min} = \pi - \cos^{-1} [ \{ ( L_3^2 + L_4^2 ) - ( L_1^2 + L_2^2 ) - 2 L_1 L_2 \} / ( 2 L_3 L_4 ) ].$$

$$[ 2.2.20 ]$$

Other Equations remain the same as in case **1**. Again using the same optimization method, $\mu_{min}$ is maximized to get $\beta_{opt}$ ( corresponding to ( $\mu_{min}$ )$_{max}$ ) and hence the desired unknown link lengths ( $L_2$, $L_3$ and $L_4$ ).

**Case 3** : [ $\theta_{24}* = \pi$ ]

In this special situation ( Fig 2.7 ), the point $C_2$ goes to infinity ( since $\angle$ $O_2FO_4$ = $\pi / 2$ ) and the circle $CR_2$ degenerates into the straight line $O_2F$.

It is obvious that, $O_2F = FN$ since $\angle$ $O_2O_4F = \angle$ $FO_4N = \theta_4* / 2$. The point $B_1$ can now be located anywhere on the line $O_2F$ above N and the crank length ( = $O_2F$ ) remains the same for all the solutions.

From Fig 2.7, we observe that

$$\beta = \pi - ( \theta_2* / 2 ), \text{ which is constant,} \qquad [ 2.2.21 ]$$

$$r_1 = L_1 \sin ( \theta_4* / 2 ) / 2 \qquad [ 2.2 22 ]$$

$$\text{and} \quad L_2 = 2 r_1 \text{ which is again constant.} \qquad [ 2.2.23 ]$$

Though the point S, in this case, can be at infinity, to achieve a compact linkage $O_2S$ is restricted to 4 $O_2F$. Thus the variable L ( = $O_2B_1$ ) is restricted in the range $2L_2$ ( = $O_2N$ ) < $O_2B_1$ < $4L_2$ ( =$O_2S$ ). After optimization it has been found that it gives the same result as the *Volmer's nomogram*.

$$\text{Now} \quad L_3 = L - L_2 \qquad [ 2.2.24 ]$$

and from $\Delta$ $O_4 B_1 D$

$$L_4 = \{ \, ( \, L \sin \beta \, )^2 + ( \, L \cos \beta - L_1 \, )^2 \, \}^{1/2}. \qquad [\, 2.2.25 \,]$$

In this case,

$$\mu_{min} = \cos^{-1} [ \, \{ \, ( \, L_3^2 + L_4^2 \, ) - ( \, L_1^2 + L_2^2 \, ) + 2 \, L_1 \, L_2 \, \} \, / \, ( \, 2 \, L_3 \, L_4 \, ) \, ].$$

$$[\, 2.2.26 \,].$$

The *golden section search method* is again used to minimize ( - $\mu_{min}$ ), but here the variable is L which varies from $2 L_2$ ( = $L_N$ ) to $4 L_2$ ( = $L_S$ ). Thus, we get $L_{opt}$ and hence $L_3$ and $L_4$.

## Case 4 : [ $\theta_{24}^* > \pi$ ]

This case is also similar to case 1 except for the change of location of the center $C_2$ of the circle $CR_2$ and as in case 2, the point S lies above the line $O_2 O_4$ ( Fig 2.8 ). Therefore,

$$r_2 = FC_2 = - r_1 \sec ( \, \theta_{24}^* / 2 \, ) \qquad [\, 2.2.27 \,]$$

and from $\Delta$ $O_2 F O_4$

$$x_{c2} = 2 \, x_{c1} - r_2 \cos ( \, \theta_4^* / 2 \, ) \qquad [\, 2.2.28 \,]$$

and

$$y_{c2} = 2 \, y_{c1} + r_2 \sin ( \, \theta_4^* / 2 \, ). \qquad [\, 2.2.29 \,]$$

Again Equation 2.2.13 gives two values of $l_n$, one for the point N and the other for the point S. Then, from Equations 2.2.11, 2.2.12 and 2.2.14 we can have $\beta_s$ and $\beta_n$.

Further, from Equation 2.2.2,

$$\mu_{min} = \cos^{-1} [ \, \{ \, ( \, L_3^2 + L_4^2 \, ) - ( \, L_1^2 + L_2^2 \, ) + 2 \, L_1 \, L_2 \, \} \, / \, ( \, 2 \, L_3 \, L_4 \, ) \, ].$$

$$[\, 2.2.30 \,]. \, .$$

Other Equations remain the same as in case 1. Then using the same optimization method $\mu_{min}$ is maximized to get $\beta_{opt}$ [ corresponding to ( $\mu_{min}$ )$_{max}$ ] and hence the desired unknown link lengths ( $L_2$, $L_3$ and $L_4$ ).

The following points regarding the Alt's construction [1] may be noted.

[ i ] If a point $B_1$ is taken outside the region NS, then the rocker has to cross the frame-line during its movement, which is not possible in crank-rocker mechanism.

[ ii ] For a given value of $\theta_4*$, the minimum and maximum values of $\theta_2*$ ( for a crank-rocker linkage to exist ) are given, respectively,by

$$( \theta_2* )_{min} = ( \pi + \theta_4* ) / 2 \quad and \quad ( \theta_2* )_{max} = ( 3 \pi + \theta_4* ) / 2.$$

[ 2.2.31 ]

In these limiting conditions, the line $O_4N$ in Alt's construction becomes tangent to $CR_2$.

**Flow chart with an illustration** - The steps used in the programming are shown in Chart 2.2. The restrictions ( given by Equation 2.2.31 ) has been used to control the input data for achieving a feasible solution. Here, the role of the function GOLD ( type, $\beta_n$, $\beta_s$, $x_{c1}$, $y_{c1}$, $x_{c2}$, $y_{c2}$ ), used in the *golden section search method* is very important. The first variable of the function selects the type ( here, type = 1 for cases 1, 2, 4 and type = 2 for case 3 ) where as ( $\beta_n$, $\beta_s$ ) define the range of the variable and the last four variables determine the transmission angle through another function MEU (type, $x_{c1}$, $y_{c1}$, $x_{c2}$, $y_{c2}$, $\beta$ ). In function GOLD ( ), a number $\tau$ ( tau = 0.618) has been used, this is known as the golden number [2]. An accuracy controlling parameter $\varepsilon$ has also been considered in the same function, which controls the accuracy of ($\mu_{min})_{max}$ and $\beta_{opt}$. We can get very accurate values just by controlling $\varepsilon$, which is really an advantage over the conventional nomogram method.

After knowing the link lengths, the function DRAW_4R( ) animates the mechanism. For verification of the results, one can use the option dead-center configuration.

Now as an illustration let us consider a dead-center problem for which

$$L_1 = 6.0 \text{ unit}, \ \theta_4* = 60^0 \text{ and } \theta_2* = 210^0.$$

The obtained final results are

$$( \mu_{\min} )_{\max} = 32.63^0,$$

$$\beta_{opt} = 46.57^0,$$

$$L_2 = 2.73 \text{ unit},$$

$$L_3 = 5.519 \text{ unit}$$

and $\quad L_4 = 6.0$ unit

The dead-center configurations of this mechanism are shown in Plot 2.3. One can also verify ( $\mu_{\min}$ )$_{\max}$ and $\beta_{opt}$ from the Volmer's nomogram ( shown in Plot 2.4 ).

[ Fig 2.1 ]

$O_2A_1 = L_2$
$A_1B_1 = L_3$
$O_2N = E$



[ Fig 2.2 ]

$O_2A = L_2$
$AB = L_3$

$O_2O_4 = L_1$
$O_2A_1 = L_2$
$A_1B_1 = L_3$
$O_4B_1 = L_4$
$O_2N = E$

[ Fig 2.3 ]



$O_2O_4 = L_1$
$O_2A = L_2$
$AB = L_3$
$O_4B = L_4$

[ Fig 2.4 ]

[ Fig 2.5 ]



[ Fig 2.6 ]
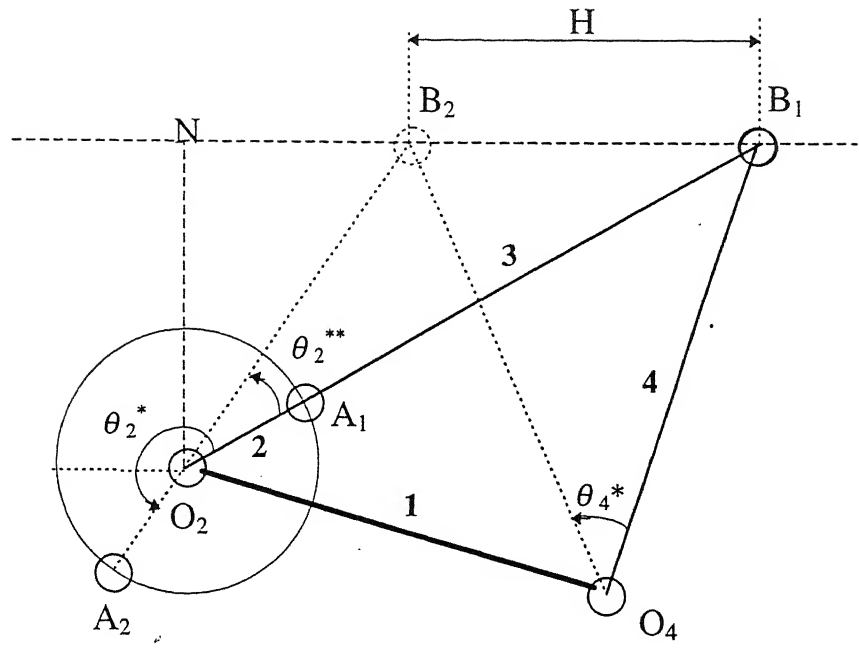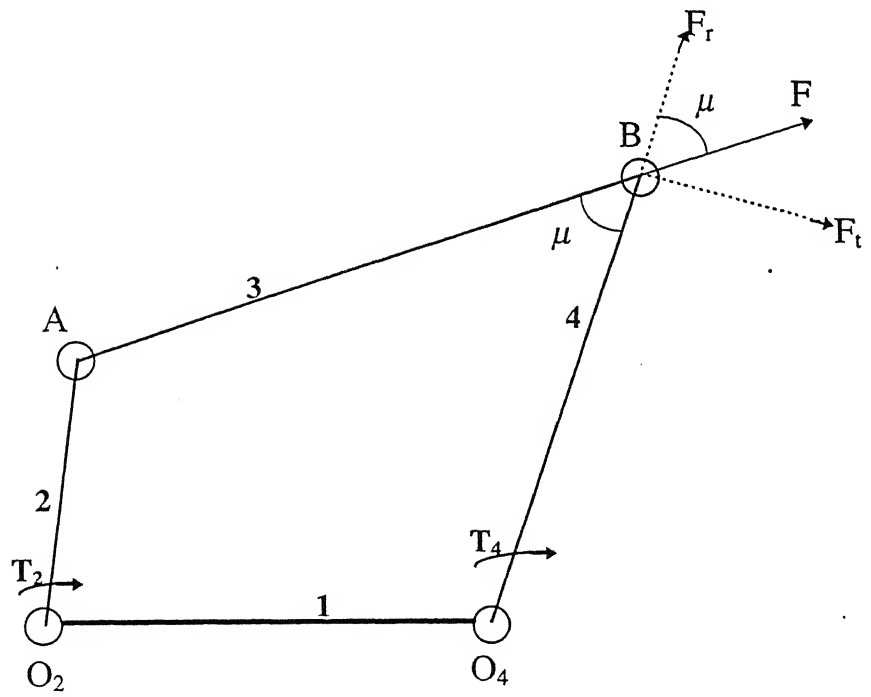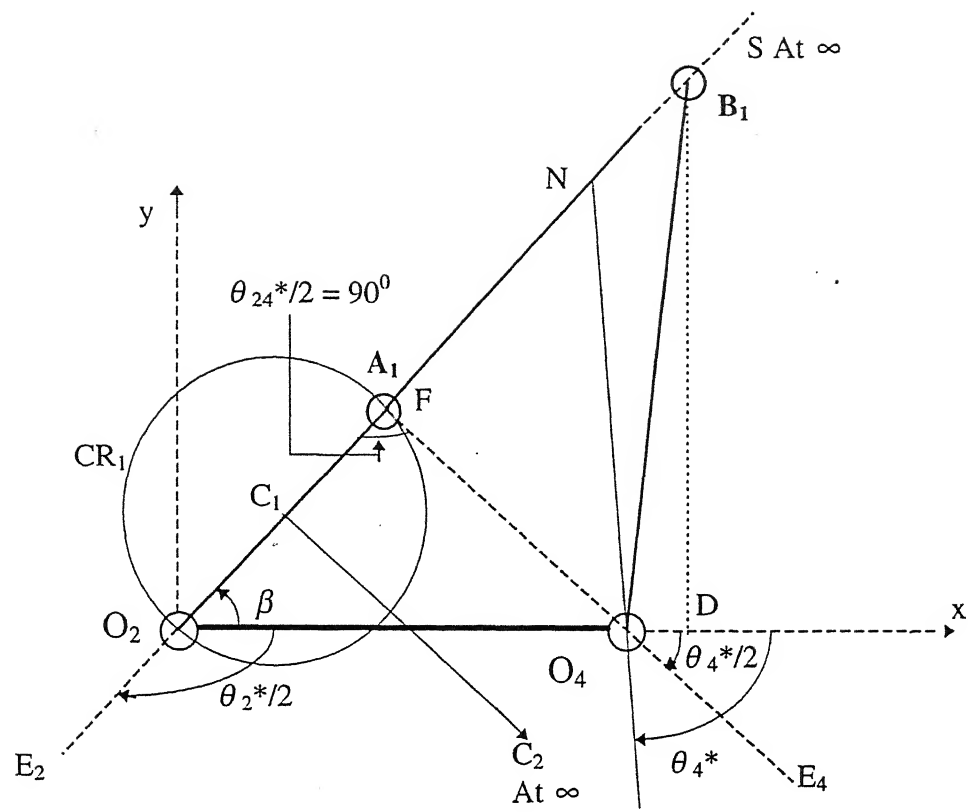
[ Fig 2.7 ]
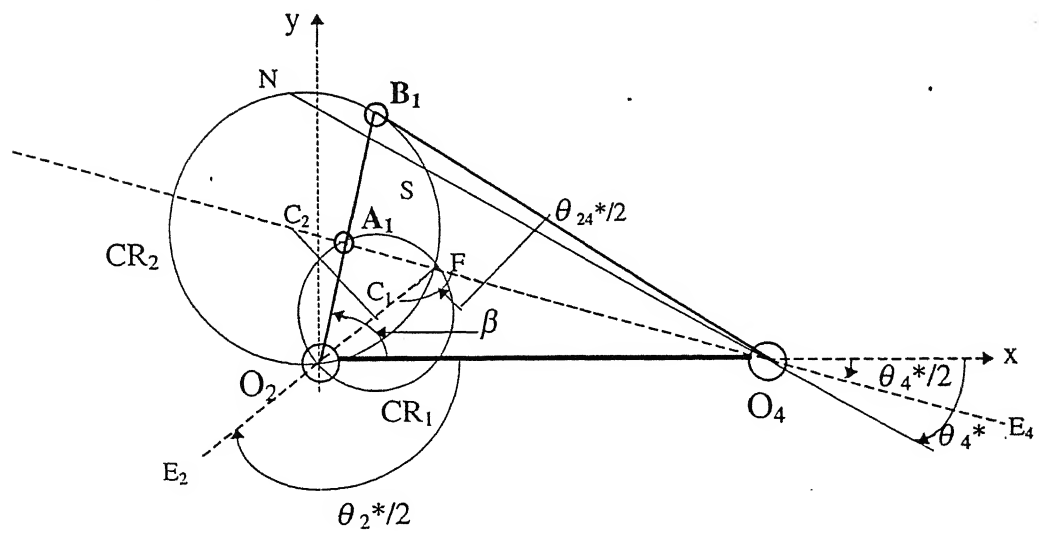


[ Fig 2.8 ]

# [ DEAD-CENTER PROBLEMS FOR 3R-1P MECHANISMS ]

INPUTS
H, $Q_{RR}$

Compute $\theta_2$**

GIVE
E or $L_3$ or $L_2$

E

$L_2$

$L_3$

No

Is
Cd1* True

Is
Cd2* True

No

Is
Cd3* True

No

Yes

Yes

Yes

Compute $L_2$ & $L_3$
from Equation. 2.1.6

Compute $L_2$ from
Equation 2.1.8

Compute $L_3$ from
Equation 2.1.11

Compute E from
Equation 2.1.10

DRAW_3R1P ( ).

* Cd1 $\rightarrow$ [ | cos( $\theta_2$** ) - ( 2 E / H ) sin ( $\theta_2$**) | $\leq$ 1 ]
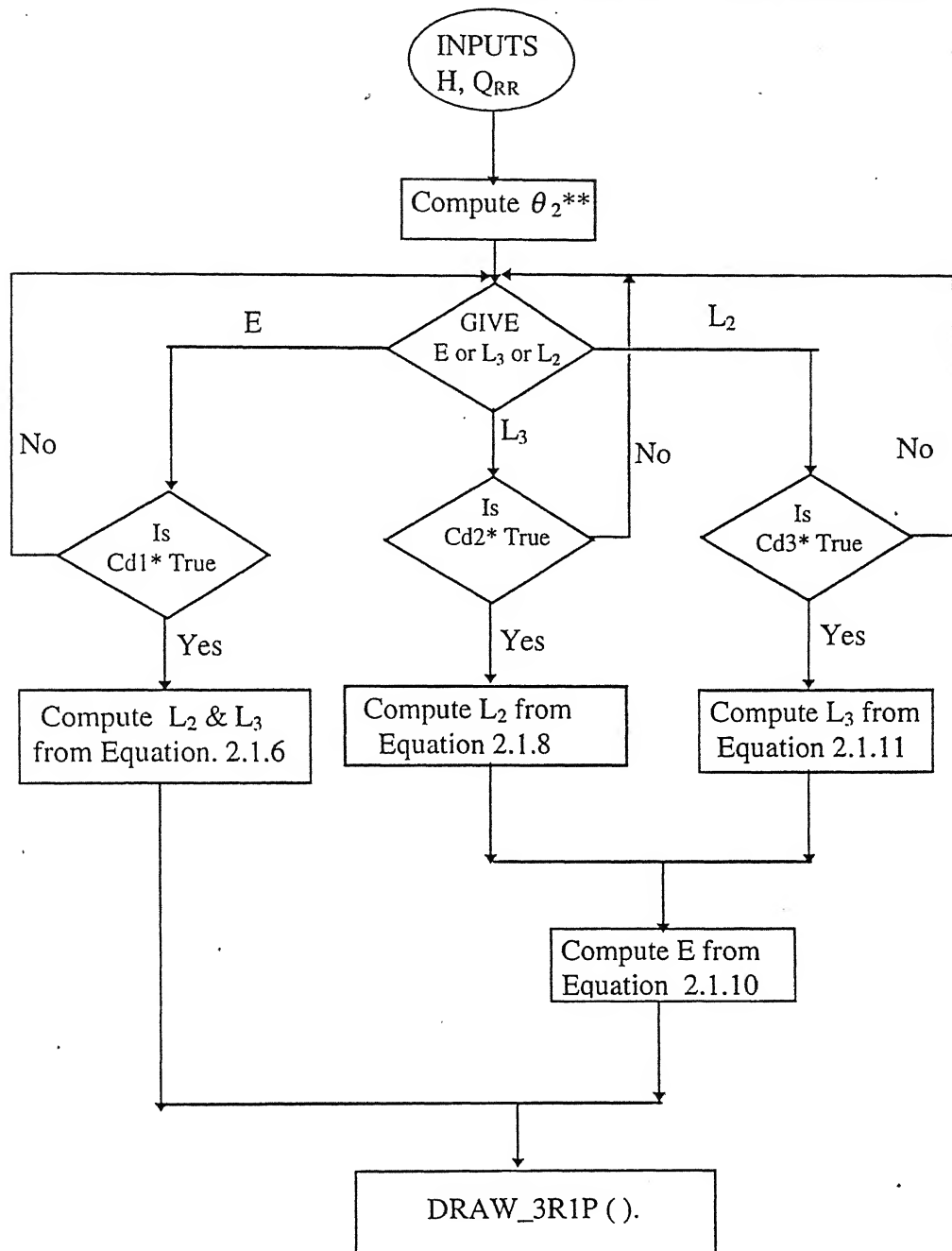
* Cd2 $\rightarrow$ [ ( $H^2/2$ ) $\geq$ $L_3^2$ ( 1 - cos( $\theta_2$** )) ]

* Cd3 $\rightarrow$ [ ( $H^2/2$ ) $\geq$ $L_2^2$ ( 1 + cos( $\theta_2$** )) ]

[ Chart 2.1 ]

# [ DEAD-CENTER PROBLEMS FOR CRANK-ROCKER MECHANISMS ]

INPUTS
$L_1$, $\theta_4^*$ & $\theta_2^*$

Is
$(\pi+\theta_4^*)/2 \le (\theta_2^*) \le (3\pi + \theta_4^*)/2$

No

No

Yes

Is $\theta_2^* - \theta_4^* = \pi$

Yes

**1, 2 & 4**

**3**

Compute $r_1$, $x_{C1}$ and $y_{C1}$ from Eqns. 2.2.5, 2.2.6 and 2.2.7 respectively

$\beta = \pi - (\theta_2^* / 2)$,
$r_1 = L_1 \sin(\theta_4^* / 2) / 2$,
$L_2 = 2 r_1$,
$L_n = 2 L_2$
**Assume** $L_s = 4 L_4$

Is
$(\theta_2^* - \theta_4^*) > \pi$

Yes

**1, 2**

No

**4.**

Compute $r_2$, $x_{C2}$ & $y_{C2}$ from Eqns. 2.2.8, 2.2.9 & 2.2.10

Compute $r_2$, $x_{C2}$ & $y_{C2}$ from Eqns. 2.2.27, 2.2.28 & 2.2.29 respy.

Replace
$\beta_n \leftarrow L_n$, $\beta_n \leftarrow L_s$,
$x_{C1} \leftarrow \beta$,
$y_{C1} \leftarrow (\theta_4^*/2)$,
$x_{C2} = 0$, $y_{C2} = 0$
type = **2,**

Is
$(\theta_2^*) \ge \pi$

No

**2**

**1**
Yes

Compute $\beta_n$, $\beta_s$
from Eqn 2.2.14

Compute $\beta_n$ from
Eqn 2.2.14, $\beta_s = 0$

type = **1,**

**10**

$$L=2(x_{c2} \cos \beta_{opt}+y_{c2} \sin \beta_{opt})$$
$$L_2=2(x_{c1} \cos \beta_{opt}+y_{c1} \sin \beta_{opt})$$
$$L_3 = L - L_2$$
$$L_4=(L^2+L_1^2-2L_1L\cos \beta_{opt})^{1/2}$$
$(\mu_{min})_{max}$ from Eqns. 2.2.2 or 2.2.3

$$L_3 = L_{opt} - L_2.$$
$$L_4=[(L_{opt} \sin \beta)^2+(-L_1+L_{opt} \cos \beta)^2]^{1/2}$$
$(\mu_{min})_{max}$ from Eqns. 2.2.2 or 2.2.3

GOLD ( type, $\beta_s$, $\beta_n$, $x_{c1}$, $y_{c1}$, $x_{c2}$, $y_{c2}$ )

If type   = 1   = 2

Print $\beta_{opt}$

$L_{opt} \leftarrow \beta_{opt}$
Print $L_{opt}$

DRAW_4R( )

[ GOLDEN SECTION SEARCH METHOD ]

$$\boxed{20}$$

$$\tau^* = 0.618,$$
$$\varepsilon^* = 1.0 \ (\text{assumed})$$
$$a_l = \beta_n - \tau \ (\beta_n - \beta_s)$$
$$b_l = \beta_s + \tau \ (\beta_n - \beta_s)$$

$$\boxed{60} \qquad
\begin{array}{l}
f(a_l) = \text{MEU}(\text{type}, x_{C1}, y_{C1}, x_{C2}, y_{C2}, \beta), \ \beta = a_l \\
f(b_l) = \text{MEU}(\text{type}, x_{C1}, y_{C1}, x_{C2}, y_{C2}, \beta), \ \beta = b_l
\end{array}
\qquad \boxed{50}$$

$> f(a_l)$      If $f(b_l)$      $< f(a_l)$

$= f(a_l)$

$$\beta_s = a_l$$
$$\beta_n = b_l$$
$$a_l = \beta_n - \tau \ (\beta_n - \beta_s)$$

$$.\ \beta_s = a_l$$
$$\beta_n = b_l$$
$$a_l = \beta_n - \tau \ (\beta_n - \beta_s)$$
$$b_l = \beta_s + \tau \ (\beta_n - \beta_s)$$

$$\beta_s = a_l$$
$$\beta_n = b_l$$
$$b_l = \beta_n + \tau \ (\beta_n - \beta_s)$$

Yes

Is $|(\beta_n - \beta_s)| > (\varepsilon \ \pi \ / \ 360)$

No

$$\boxed{30}$$

$$L = 2(x_{c2} \cos \beta + y_{c2} \sin \beta)$$
$$L_2 = 2(x_{c1} \cos \beta + y_{c1} \sin \beta)$$
$$L_3 = L - L_2$$
$$L_4 = (L^2 + L_1^2 - 2L_1 L \cos \beta)^{1/2}$$

$$L_3 = L - L_2.$$
$$L_4 = [(L \sin \beta)^2 + (-L_1 + L \cos \beta)^2]^{1/2}$$

If type     = 1     = 2

(- $\mu_{min}$) from Equation 2.2.2 or 2.2.3

50

60

[ Chart 2.2 ]

L2 = 2.8629
L3 = 4.494
E = 1

A Grashofian

Lmax / Lmin = 1.57

Min. Transmission
Angle = 30.73 degree

Xmin of Slider = 1.289

Xmax of Slider = 7.289

[ Plot 2.1 ]

L2 = 2.7307
L3 = 5.5222
E = 1.9198

A Grashofian

Lmax / Lmin = 2.022

Min transmission Angle
= 32.63 degree

Xmin of Slider = 2.027

Xmax of Slider = 8.026

[ Plot 2.2 ]

L1 = 6.0
L2 = 2.7312
L3 = 5.5189
L4 = 6.0

A Grashofian
A Crank-rocker

Lmax / Lmin = 2.197

Min transmission Angle
= 32.63 degree

Min. Rocker Angle =
93.13 degree ( CCW )

Max. Rocker Angle =
153.1 degree ( CCW )

[ Plot 2.3 ]

**Volmer's Nomogram[10]**

[ Plot 2.4 ]

# 3. APPROACH FOR PATH AND FUNCTION GENERATION

In this chapter we shall discuss a *least square technique* for synthesizing 4R mechanisms for path generation and function generation ( or coordinated input-output ) problems. This approach is generally used for more than three design positions.

## 3.1 . PATH GENERATION

Here the coordinates of some points on the specified path are available. These points may or may not be coordinated with the input angles. If the coordinated input angles are not specified then one can assume these angles which are required in this method. Now we have to synthesize a 4R mechanism such that its coupler point approximately passes through the specified points.

Figure 3.1 shows a 4R mechanism, $O_2ABCO_4$ with unknown design parameters $R_1$, $R_2$, $R_3$, $R_4$, $R_5$, $R_6$, $\hat{\alpha}$, $\hat{\beta}$, $\phi$ and $\psi$ out of which $\alpha$ and $\beta$ are assumed initially. Suppose there are n points through which the coupler point C has to pass and the coupler point at the $i^{th}$ position is specified by its polar coordinates ( $r_i$, $\delta_i$ ) with the coordinated input represented by $\theta_{2i}$.

Now, writing the loop-closure equations for the loop $OO_2AC_1O$, we get .

$$R_1 \cos \hat{\alpha} + R_2 \cos \theta_{2i} + R_3 \cos \psi_i = r_i \cos \delta_i \qquad [\ 3.1.1\ ]$$

and

$$R_1 \sin \hat{\alpha} + R_2 \sin \theta_{2i} + R_3 \sin \psi_i = r_i \sin \delta_i . \qquad [\ 3.1.2\ ]$$

Similarly, for the loop $OO_4BC_1O$, we have

$$R_4 \cos \hat{\beta} + R_5 \cos \theta_{4i} + R_6 \cos \phi_i = r_i \cos \delta_i \qquad [\ 3.1.3\ ]$$

and

$$R_4 \sin \hat{\beta} + R_5 \sin \theta_{4i} + R_6 \sin \phi_i = r_i \sin \delta_i. \qquad [3.1.4]$$

Eliminating $\psi_i$ from Equations 3.1.1 and 3.1.2, we get

$$( R_1 \cos \hat{\alpha} + R_2 \cos \theta_{2i} - r_i \cos \delta_i )^2 + ( R_1 \sin \hat{\alpha} + R_2 \sin \theta_{2i} - r_i \sin \delta_i )^2 = R_3^2$$

or

$$r_i^2 + R_2^2 + R_1^2 + 2 R_1 R_2 \cos ( \theta_{2i} - \hat{\alpha} ) - 2 R_1 r_i \cos ( \hat{\alpha} - \delta_i )$$
$$- 2 R_2 r_i \cos ( \theta_{2i} - \delta_i ) = R_3^2$$

or

$$R_1 [ 2 r_i \cos ( \hat{\alpha} - \delta_i ) ] + R_2 [ 2 r_i \cos ( \theta_{2i} - \delta_i ) ] + ( R_3^2 - R_2^2 - R_1^2 )$$
$$= r_i^2 + R_1 R_2 [ 2 \cos ( \theta_{2i} - \hat{\alpha} ) ]. \qquad [3.1.5]$$

Similarly, eliminating $\theta_{4i}$ from Equations 3.1.3 and 3.1.4, we get

$$R_4 [ 2 r_i \cos ( \hat{\beta} - \delta_i ) ] + R_6 [ 2 r_i \cos ( \phi_i - \delta_i ) ] + ( R_5^2 - R_4^2 - R_6^2 )$$
$$= r_i^2 + R_4 R_6 [ 2 \cos ( \phi_i - \hat{\beta} ) ]. \qquad [3.1.6]$$

Substituting

$$
\begin{array}{llll}
D_1 = R_1, & & D_5 = R_4, & \\
D_2 = R_2, & & D_6 = R_6, & \\
D_3 = R_3^2 - R_2^2 - R_1^2, & & D_7 = R_5^2 - R_4^2 - R_6^2 & \qquad [3.1.7] \\
D_4 = R_1 R_2, & \text{and} & D_8 = R_4 R_6 &
\end{array}
$$

Equations 3.1.5 and 3.1.6 are linearised in the design parameters $D_i$'s ( i = 1 to 8 ) as

$$D_1 [ 2 r_i \cos ( \hat{\alpha} - \delta_i ) ] + D_2 [ 2 r_i \cos ( \theta_{2i} - \delta_i ) ] + D_3$$
$$- r_i^2 - D_4 [ 2 \cos ( \theta_{2i} - \hat{\alpha} ) ] = 0 \qquad [3.1.8]$$

and

$$D_5 [ 2 r_i \cos ( \hat{\beta} - \delta_i ) ] + D_6 [ 2 r_i \cos ( \phi_i - \delta_i ) ] + D_7$$

$$- r_i^2 - D_8 [ 2 \cos ( \phi_i - \hat{\beta} ) ] = 0. \qquad [ 3.1.9 ]$$

The design parameters are nothing but functions of the link lengths. If the coupler point passes exactly through the prescribed points at the desired input angles then Equations 3.1.8 and 3.1.9 are perfectly satisfied. Here, only $r_i$, $\delta_i$ and $\theta_{2i}$ ( i = 1 to n ) are available, but while solving Equations 3.1.8 and 3.1.9, it requires some assumptions

( say $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$ ). At this stage, it must be appreciated that Equations 3.1.8 and 3.1.9 are not satisfied exactly. Now let $e_{1i}$ and $e_{2i}$ represent the error in the loop-closure equations 3.1.8 and 3.1.9, respectively, (at the $i^{th}$ design position ). Thus,

$$e_{1i} = D_1 [ 2 r_i \cos ( \hat{\alpha} - \delta_i ) ] + D_2 [ 2 r_i \cos ( \theta_{2i} - \delta_i ) ] + D_3$$

$$- r_i^2 - D_4 [ 2 \cos ( \theta_{2i} - \hat{\alpha} ) ] \qquad [ 3.1.10 ]$$

and

$$e_{2i} = D_5 [ 2 r_i \cos ( \hat{\beta} - \delta_i ) ] + D_6 [ 2 r_i \cos ( \phi_i - \delta_i ) ] + D_7$$

$$- r_i^2 - D_8 [ 2 \cos ( \phi_i - \hat{\beta} ) ]. \qquad [ 3.1.11 ]$$

It is obvious from Equation 3.1.7 that

$$D_4 = D_1 D_2 \quad \text{and} \quad D_8 = D_5 D_6.$$

Thus $D_4$ and $D_8$ make the Equations 3.1.10 and 3.1.11 non-linear. Let us first solve Equation 3.1.10 as explained below.

Let

$$D_4 = D_1 D_2 = \lambda \qquad\qquad [ 3.1.12 ]$$

and $\quad D_p = l_p + \lambda m_p$, for p = 1, 2, 3. $\qquad\qquad [ 3.1.13 ]$

From Equations 3.1.12 and 3.1.13

$$\lambda = ( l_1 + \lambda \ m_1 ) ( l_2 + \lambda \ m_2 ).$$

[3.1.14]

Therefore, from equation 3.1.10

$$e_{1i} = e_{1i,l} + \lambda \ e_{1i,m} ,$$

[3.1.15]

where

$$e_{1i,l} = l_1 [ 2 \ r_i \cos (\hat{\alpha} - \delta_i ) ] + l_2 [ 2 \ r_i \cos ( \theta_{2i} - \delta_i ) ] + l_3 - r_i^2$$

and

$$e_{1i,m} = m_1 [ 2 \ r_i \cos (\hat{\alpha} - \delta_i ) ] + m_2 [ 2 \ r_i \cos ( \theta_{2i} - \delta_i ) ] + m_3 - 2 \cos ( \theta_{2i} - \hat{\alpha} ).$$

In the ideal case, $e_{1i} = 0$. To achieve this we would like to get l's and m's such that $e_{1i,l} = 0$ and $e_{1i,m} = 0$.

Similarly, for solving Equation 3.1.11,

let

$$D_8 = D_5 D_6 = \eta$$

[3.1.16]

and

$$D_p = u_p + \eta \ v_p , \text{ for } p = 5, 6, 7.$$

[3.1.17]

From Equations 3.1.16 and 3.1.17, we get

$$\eta = ( u_1 + \eta \ v_1 ) ( u_2 + \eta \ v_2 ).$$

[3.1.18]

Therefore, from equation 3.1.11

$$e_{2i} = e_{2i,u} + \lambda \ e_{2i,v} ,$$

[3.1.19]

where

$$e_{2i,u} = u_1 [ 2 \ r_i \cos (\hat{\beta} - \delta_i ) ] + u_2 [ 2 \ r_i \cos ( \phi_i - \delta_i ) ] + u_3 - r_i^2$$

and

$$e_{2i,v} = v_1 [ 2 r_i \cos (\hat{\beta} - \delta_i)] + v_2 [ 2 r_i \cos (\phi_i - \delta_i)] + v_3 - 2 \cos (\phi_i - \hat{\beta}).$$

Again in the ideal case $e_{2i} = 0$. To achieve this we would like to get u's and v's such that $e_{2i,u} = 0$ and $e_{2i,v} = 0$.

Using the least square technique, now we shall minimize the overall error ( e ), which is expressed as

$$e = \sum_{i=1}^{n} wt_i ( e_{1i,l}^2 + e_{1i,m}^2 + e_{2i,u}^2 + e_{2i,v}^2 ), \qquad [3.1.20]$$

where the *error functions* are weighted with $wt_i$ for the $i^{th}$ position.
It helps the designer to minimize the error more at the positions where he / she needs to do so. In other words, after some number of iterations, the designer can obtain a better solution with the help of these weighting functions $wt_i$. In the first iteration, $wt_i$ are taken as unity for all i's and then it is checked at which design positions errors are more than acceptable. Thereafter, the $wt_i$'s are modified accordingly. After the first iteration, we have taken $wt_i$ equal to the structural error at the $i^{th}$ position. By iteration with these weighting functions the user can minimize the structural error at some critical points.

Now, for the minimum value of e, we set the first partial derivatives to zero. Thus, we get

$$\frac{\partial}{\partial l_1} [ \sum_{i=1}^{n} wt_i \, e_{1i,l}^2 ] = \frac{\partial}{\partial l_2} [ \sum_{i=1}^{n} wt_i \, e_{1i,l}^2 ] = \frac{\partial}{\partial l_3} [ \sum_{i=1}^{n} wt_i e_{1i,l}^2 ] = 0,$$

$$[3.1.21a]$$

$$\frac{\partial}{\partial m_1} [ \sum_{i=1}^{n} wt_i \, e_{1i,m}^2 ] = \frac{\partial}{\partial m_2} [ \sum_{i=1}^{n} wt_i \, e_{1i,m}^2 ] = \frac{\partial}{\partial m_3} [ \sum_{i=1}^{n} wt_i \, e_{1i,m}^2 ] = 0,$$

$$[3.1.21b]$$

$$\frac{\partial}{\partial u_1} \left[ \sum_{i=1}^{n} wt_i \, e_{2i,u}{}^2 \right] = \frac{\partial}{\partial u_2} \left[ \sum_{i=1}^{n} wt_i \, e_{2i,u}{}^2 \right] = \frac{\partial}{\partial u_3} \left[ \sum_{i=1}^{n} wt_i \, e_{2i,u}{}^2 \right] = 0$$

$$[\, 3.1.21c \,]$$

and

$$\frac{\partial}{\partial v_1} \left[ \sum_{i=1}^{n} wt_i \, e_{2i,v}{}^2 \right] = \frac{\partial}{\partial v_2} \left[ \sum_{i=1}^{n} wti \, e_{2i,v}{}^2 \right] = \frac{\partial}{\partial v_3} \left[ \sum_{i=1}^{n} wti \, e_{2i,v}{}^2 \right] = 0.$$

$$[\, 3.1.21d \,]$$

Now, onwards the summation indices i and n are omitted.

Equation 3.1.21a gives

$$l_1 \sum \left[ wt_i \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \}^2 \right]$$

$$+ l_2 \sum \left[ wt_i \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \} \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \} \right]$$

$$+ l_3 \sum \left[ wt_i \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \} \right]$$

$$= \sum \left[ wt_i \, r_i{}^2 \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \} \right],$$

$$[\, 3.1.22a \,]$$

$$l_1 \sum \left[ wt_i \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \} \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \} \right]$$

$$+ l_2 \sum \left[ wt_i \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \}^2 \right]$$

$$+ l_3 \sum \left[ wt_i \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \} \right]$$

$$= \sum \left[ wt_i \, r_i{}^2 \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \} \right]$$

$$[\, 3.1.22b \,]$$

and

$$l_1 \sum \left[ wt_i \{ 2\, r_i \cos (\hat{\alpha} - \delta_i) \} \right]$$

$$+ l_2 \sum \left[ wt_i \{ 2\, r_i \cos (\theta_{2i} - \delta_i) \} \right]$$

$$+ l_3 \sum \left[ wt_i \right]$$

$$= \sum \left[ wt_i \, r_i{}^2 \right]. \qquad [\, 3.1.22c \,]$$

Thus, these three equations can be solved by using *Cramer's rule* to determine unknowns $l_1$, $l_2$ and $l_3$.

In a similar manner, Equation 3.1.21b gives

$$m_1 \sum [wt_i \{2 r_i \cos(\hat{\alpha} - \delta_i)\}^2]$$

$$+ m_2 \sum [wt_i \{2 r_i \cos(\theta_{2i} - \delta_i)\} \{2 r_i \cos(\hat{\alpha} - \delta_i)\}]$$

$$+ m_3 \sum [wt_i \{2 r_i \cos(\hat{\alpha} - \delta_i)\}]$$

$$= \sum [wt_i \{2 \cos(\theta_{2i} - \hat{\alpha})\} \{2 r_i \cos(\hat{\alpha} - \delta_i)\}],$$

$$[3.1.23a]$$

$$m_1 \sum [wt_i \{2 r_i \cos(\hat{\alpha} - \delta_i)\} \{2 r_i \cos(\theta_{2i} - \delta_i)\}]$$

$$+ m_2 \sum [wt_i \{2 r_i \cos(\theta_{2i} - \delta_i)\}^2]$$

$$+ m_3 \sum [wt_i] \{2 r_i \cos(\theta_{2i} - \delta_i)\}$$

$$= \sum [wt_i \{2 \cos(\theta_{2i} - \hat{\alpha})\} \{2 r_i \cos(\theta_{2i} - \delta_i)\}]$$

$$[3.1.23b]$$

and

$$m_1 \sum [wt_i \{2 r_i \cos(\hat{\alpha} - \delta_i)\}]$$

$$+ m_2 \sum [wt_i \{2 r_i \cos(\theta_{2i} - \delta_i)\}]$$

$$+ m_3 \sum [wt_i]$$

$$= \sum [wt_i \{2 \cos(\theta_{2i} - \hat{\alpha})\}]. \qquad [3.1.23c]$$

Again these three equations can be solved by using *Cramer's rule* to determine $m_1$, $m_2$ and $m_3$. Once $l_1$, $l_2$, $l_3$ and $m_1$, $m_2$, $m_3$ are known, we can determine $\lambda$ ( from Equation 3.1.14 ), and then $D_1$, $D_2$ and $D_3$ ( from Equation 3.1.13 ) and hence $R_1$, $R_2$ and $R_3$ ( from Equation 3.1.7 ).

Now we can determine $\psi_i$'s, i=1 to n, by using Equation 3.1.1 or 3.1.2.

Assuming a suitable value for $\phi_1$, we can determine $\phi_i$ ( for i = 2 to n ) by using the following relations.

$$\phi_i = \phi_1 + (\phi_i - \psi_i), (\text{ for } i = 2 \text{ to } n)$$

[ 3.1.24 ]

Thus, we have two values of $\lambda$ ( say $\lambda_1$ and $\lambda_2$ ) and correspondingly two values of $R_1$, $R_2$ and $R_3$ ( say $R_1[1]$, $R_2[1]$, $R_3[1]$ and $R_1[2]$, $R_2[2]$, $R_3[2]$ ) and hence two values of $\phi_i$'s ( for $i = 2$ to $n$ ).

In the same manner, Equations 3.1.21c and 3.1.21d give

$$u_1 \sum \left[ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}^2 \right]$$
$$+ u_2 \sum \left[ wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \} \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \right]$$
$$+ u_3 \sum \left[ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \right]$$
$$= \sum \left[ wt_i r_i^2 \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \right],$$

[ 3.1.25a ]

$$u_1 \sum \left[ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \{ 2 r_i \cos (\phi_i - \delta_i) \} \right]$$
$$+ u_2 \sum \left[ wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \}^2 \right]$$
$$+ u_3 \sum \left[ wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \} \right]$$
$$= \sum \left[ wt_i r_i^2 \{ 2 r_i \cos (\phi_i - \delta_i) \} \right],$$

[ 3.1.25b ]

$$u_1 \sum \left[ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \right]$$
$$+ u_2 \sum \left[ wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \} \right]$$
$$+ u_3 \sum \left[ wt_i \right]$$
$$= \sum \left[ wt_i r_i^2 \right],$$

[ 3.1.25c ]

$$v_1 \sum [ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}^2 ]$$

$$+ v_2 \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \delta_i) \} \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} ]$$

$$+ v_3 \sum [ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} ]$$

$$= \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \hat{\beta} ) \} \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} ],$$

$$[ 3.1.26a ]$$

$$v_1 \sum [ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \{ 2 r_i \cos ( \phi_i - \delta_i) \} ]$$

$$+ v_2 \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \delta_i) \}^2 ]$$

$$+ v_3 \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \delta_i) \} ]$$

$$= \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \hat{\beta} ) \} \{ 2 r_i \cos ( \phi_i - \delta_i) \} ]$$

$$[ 3.1.26b ]$$

and

$$v_1 \sum [ wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} ]$$

$$+ v_2 \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \delta_i) \} ]$$

$$+ v_3 \sum [ wt_i ]$$

$$= \sum [ wt_i \{ 2 r_i \cos ( \phi_i - \hat{\beta} ) \} ].$$

$$[ 3.1.26c ]$$

Again, from Equations 3.1.25a, 3.1.25b and 3.1.25c we determine $u_1$, $u_2$ and $u_3$. Similarly $v_1$, $v_2$ and $v_3$ are obtained from Equations 3.1.26a, 3.1.26b and 3.1.26c. Therefore we get $\eta$ ( from Equation 3.1.18 ), and then $D_4$, $D_5$ and $D_6$ ( from Equation 3.1.17 ) and hence $R_4$, $R_5$ and $R_6$ ( from Equation 3.1.7 ). There are two values of $\phi_i$'s ( for i = 2 to n ) and each value of $\phi_i$'s ( for i = 2 to n ) gives two values $\eta$ [ say the first value of $\phi_i$'s ( for i = 2 to n ) gives $\eta_{11}$, $\eta_{12}$ and the second value of $\phi_i$'s ( for i = 2 to n ) gives $\eta_{21}$, $\eta_{22}$ ]. Hence there are four values $\eta$ and therefore, we get four values of $R_4$, $R_5$ and $R_6$, which are denoted as

$R_4[1][1]$, $R_4[1][2]$, $R_4[2][1]$, $R_4[2][2]$,

$R_5[1][1]$, $R_5[1][2]$, $R_5[2][1]$, $R_5[2][2]$

and

$R_6[1][1]$, $R_6[1][2]$, $R_6[2][1]$, $R_6[2][2]$.

Hence, we have four solutions as

[i] $R_1[1]$, $R_2[1]$, $R_3[1]$, $R_4[1][1]$, $R_5[1][1]$, $R_6[1][1]$ $\hat{\alpha}$, $\hat{\beta}$, $\phi_1$,

[ii] $R_1[1]$, $R_2[1]$, $R_3[1]$, $R_4[1][2]$, $R_5[1][2]$, $R_6[1][2]$ $\hat{\alpha}$, $\hat{\beta}$, $\phi_1$,

[iii] $R_1[2]$, $R_2[2]$, $R_3[2]$, $R_4[2][1]$, $R_5[2][1]$, $R_6[2][1]$ $\hat{\alpha}$, $\hat{\beta}$, $\phi_1$

and

[iv] $R_1[2]$, $R_2[2]$, $R_3[2]$, $R_4[2][2]$, $R_5[2][2]$, $R_6[2][2]$ $\hat{\alpha}$, $\hat{\beta}$, $\phi_1$.

Now, the parameters ( $R_1$, $R_2$, $R_3$, $R_4$, $R_5$, $R_6$, $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$ ) can be converted into the link lengths ( $L_1$, $L_2$, $L_3$ and $L_4$ ) and the polar coordinates ( R, $\gamma$ ) of the coupler point, such that

$$L_1 = O_2O_4 = [ ( R_4 \cos\hat{\beta} - R_1 \cos\hat{\alpha} )^2 + ( R_4 \sin\hat{\beta} - R_1 \sin\hat{\alpha} )^2 ]^{1/2},$$

$$L_2 = O_2A = R_2,$$

$$L_3 = AB = [ R_6^2 + R_3^2 - 2 R_6 R_3 \cos ( \phi_1 - \psi_1 ) ]^{1/2},$$

$$L_4 = O_4B = R_5,$$

$$R = AC = R_3,$$

and $\quad \gamma = \angle CAB = \sin^{-1}[ R_6 \sin ( \phi_1 - \psi_1 ) / L_3 ].$

[ 3.1.27 ]

These new parameters ( $L_1$, $L_2$, $L_3$, $L_4$, R and $\gamma$ ) are used in the animation of a 4R mechanism for path generation.

Hence, we can find $L_1[k]$, $L_2[k]$, $L_3[k]$, $L_4[k]$, $R[k]$ and $\gamma[k]$ for each $k^{th}$ solution. It is not guaranteed that all the above solutions give acceptable results. It mainly depends upon the assumed values of $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$ needed to solve the equations. By changing these values one can arrive at better solutions.

**Flow Chart** - Chart 3.1 shows the steps used in the programming. Here are some mathematical bridges ( Quad1, Sqrt1, Quad2, Sqrt2, see in Chart 3.1 ), without getting these positive one can not move further. These expressions are controlled by changing the assumed values of $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$. Some bridges are being controlled automatically and others need manual controlling. Regarding the selection of $\hat{\alpha}$ and $\hat{\beta}$, one should choose the parameters such that most of the specified points lie near the perpendicular to the line $O_2O_4$ and to get a crank-rocker or a double-rocker mechanism, the specified points must lie on one side of the line $O_2O_4$.

The present software can also handle some constraints like

[a] the ratio ( $L_{max}$ / $L_{min}$ ), where $L_{max}$ and $L_{min}$ are the maximum and minimum values of link lengths of the mechanism,

[b] the designed mechanism must be a crank-rocker and

[c] the structural error at a specified point must be zero i.e, the coupler point must pass through a specific point on the path.

The ratio $L_{max}$ / $L_{min}$ is controlled by $\hat{\beta}$ whereas a crank-rocker can be ensured by $\phi_1$. Here, to minimize the structural error, the least square error function is weighted by the distance between the specified and generated points at each prescribed $\theta_2$. Obviously some trials are necessary to arrive at an acceptable solution. Here some *steps* are given, following which one can easily run the program quickly to get a good result.

[ i ] Give n, $(x_i, y_i)$ or ( $r_i$, $\delta_i$ ) and $\theta_{2i}$.

[ ii ] Select any one of the solutions ( 1, 2, 3 and 4 ) and assume ( $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$ ) as default values. Initially the constraints are left out.

[ iii ] Quad1 is automatically controlled, but controlling of Sqrt1 may require a manual change of $\hat{\alpha}$. Similarly, Quad1 is controlled automatically but Sqrt2 is controlled by changing $\hat{\beta}$ manually. The program itself takes care of the constraints on the ratio ( $L_{max}$ / $L_{min}$ ) and the crank-rocker compulsion.

[ iv ] Check the other three solutions and choose the best one. Now, if the structural errors are large, then go to step [ii] and change the assumptions keeping in mind the previous results and the assumptions made. After two to three trials, one can guess the suitable values of $\hat{\alpha}$, $\hat{\beta}$ and $\phi_1$. The variation of $\phi_1$ can help to reach a good solution.

[ v ] For better results, choose an option of weighting the error function. Weighting on the error functions are possible only in the case of crank-rocker and double-rocker mechanisms. One should check the other three solutions if one solution exists.

If instead of prescribed points, a path is given, then start solving the problem by considering 6 points on the path at equal steps of $\theta_2$. Then increase the number of points to 8, 10 and 12. The present software considers a maximum of twelve points but one can replace it by any number. However, care should be taken regarding the memory and the speed of the computing system.

**An Illustration** - Let us consider a path generated by a crank-rocker mechanism ( shown in Plot 3.1 ) with

$$L_1 = 2 \text{ unit,}$$
$$L_2 = 1 \text{ unit,}$$
$$L_3 = 2 \text{ unit,}$$
$$L_4 = 2 \text{ unit,}$$
$$R = 3 \text{ unit}$$
and $\gamma = 120^0$.

Number of points selected = 12, which are

| Point | x — coordinate | y — coordinate | $\theta_2$ |
|---|---|---|---|
| 1 | -1.8906 | -0.8028 | 0.0 |
| 2 | -2.0701 | 1.1157 | 30.0 |
| 3 | -2.2042 | 2.1651 | 60.0 |
| 4 | -2.5835 | 2.525 | 90.0 |
| 5 | -3.0845 | 2.3894 | 120.0 |
| 6 | -3.5495 | 1.8414 | 150.0 |
| 7 | -3.8435 | 0.9564 | 180.0 |
| 8 | -3.8451 | -0.1465 | 210.0 |
| 9 | -3.4730 | -1.2682 | 240.0 |
| 10 | -2.7701 | -2.1518 | 270.0 |
| 11 | -1.9771 | -2.5584 | 300.0 |
| 12 | -1.5689 | -2.2525 | 330.0 |

For this problem, assuming $\hat{\alpha}$, $\hat{\beta}$, $\phi_1$ and $L_{max}/L_{min}$ as default values ($\hat{\alpha} = 56^0$, $\hat{\beta} = 354^0$, $\phi_1 = 166^0$ and $L_{max}/L_{min} = 20$) we get the following results.

| Solution no. | $\hat{\alpha}$ | $\hat{\beta}$ | $\phi_1$ | $L_{max}/L_{min}$ | Type of mechanism | ( NP )Number of specified points through which the coupler point approximately passes. |
|---|---|---|---|---|---|---|
| 1 | 56 | 354 | 166 | 19.62 | Crank - rocker | 2 |
| 2 | 56 | 354 | 166 | 1.497 | Non Grashofian | 6 |
| 3 | 56 | 354 | 166 | 10.13 | Non Grashofian | 2 |
| 4 | 56 | 354 | 166 | 2.289 | Non Grashofian | 8 |

In the next iteration, an attempt is made to get a crank-rocker mechanism for solution no. 4. The final results turn out as $L_{max} / L_{min} = 1.968$ and NP = 4 for a crank-rocker mechanism. Then for a new set of assumptions ($\hat{\alpha} = 50^0$, $\hat{\beta} = 50^0$, $\phi_1 = 90^0$ and $L_{max} / L_{min} = 20$), the following results are obtained.

| Solution no. | $\hat{\alpha}$ | $\hat{\beta}$ | $\phi_1$ | $L_{max} / L_{min}$ | Type of mechanism | (NP)Number . of specified points through which the coupler point approximately passes. |
|---|---|---|---|---|---|---|
| 1 | 50 | 50 | 90 | 13.56 | Crank - rocker | 5 |
| 2 | 50 | 50 | 90 | 1.852 | Non Grashofian | 3 |
| 3 | 50 | 50 | 90 | 13.07 | Crank - rocker | 4 |
| 4 | 50 | 50 | 90 | 5.366 | Crank - rocker | 9 |

Out of all the four solutions, the fourth one can be selected ( see Plot 3.2 ). To observe the accuracy of the results, X_error ( % ) and Y_error ( % ) are introduced. These errors, at an $i^{th}$ position ( i.e. corresponding to $\theta_{2i}$ ), are defined as

$$( \% ) \, X\_error \, [i] = [ \{ ( x_i )_g - ( x_i )_s \} \, 100 \, / \, \{ ( x_{max} )_s - ( x_{min} )_s \} ]$$
and
$$( \% ) \, Y\_error \, [i] = [ \{ ( y_i )_g - ( y_i )_s \} \, 100 \, / \, \{ ( y_{max} )_s - ( y_{min} )_s \} ],$$

where
$i = 1$ to n ( no of specified points ),

suffixes 'g' and 's' denote the generated and specified points,

$x_{max}$ = the maximum value of $x_i$ ( x _ coordinate of the point at the $i^{th}$ position ),

$x_{min}$ = the minimum value of $x_i$,

$y_{max}$ = the maximum value of $y_i$ ( y _ coordinate of the point at the $i^{th}$ position )

and $y_{min}$ = the minimum value of $y_i$.

To reduce the errors near the points 3 and 4 ( as expected ) weighting of the error function has been considered and after the first iteration a better solution (regarding those two points ) is obtained ( see Plot 3.3 ). The errors at all the points corresponding to $\theta_{2i}$ ( i = 1 to n ), before and after weighting the error functions, are given below.

| Point | $\theta_2$ | Before weighting the error function | | After weighting the error function | |
|-------|-----------|-------------------|-------------------|-------------------|-------------------|
| | | ( % ) X_error | ( % ) Y_error | ( % ) X_error | ( % ) Y_error |
| 1 | 0 | 14.82 | -15.19 | 6.22 | -7.83 |
| 2 | 30 | -2.65 | -14.99 | -2.50 | -8.11 |
| *3 | 60 | 21.41 | 14.21 | 14.11 | 10.30 |
| *4 | 90 | 13.93 | 8.67 | 7.76 | 5.23 |
| 5 | 120 | 3.029 | -2.19 | -1.24 | -0.96 |
| 6 | 150 | -2.70 | -2.58 | -5.27 | -5.38 |
| 7 | 180 | -3.07 | -4.73 | -4.20 | -6.99 |
| 8 | 210 | -0.71 | -3.62 | -0.86 | -5.10 |
| 9 | 240 | -0.0002 | 0.0065 | 0.14 | 0.43 |
| 10 | 270 | -3.28 | 3.925 | -3.92 | 4.79 |
| 11 | 300 | -7.34 | 5.414 | -10.1 | 7.85 |
| 12 | 300 | -1.63 | 1.039 | 7.83 | 5.485 |

The errors at the other points ( e.g. 11 and 12 ) are now more. So applying weights is more useful when the structural error at some points are critical for a particular application.

## 3.2 . FUNCTION GENERATION

If a 4R mechanism has to generate a particular function (which is given ), with more than three precision positions, then the optimization method is more useful regarding the overall minimization of the error.

Let $\theta_{2\text{-}0i}$ and $\theta_{4\text{-}0i}$ be the input and output angles, respectively at the $i^{th}$ position as shown in Fig 3.2. Now let us assume the initial positions of the input and output links, which are at angles $\theta_{2\text{-}0}$ and $\theta_{4\text{-}0}$, respectively ( Fig 3.2 ). Therefore, from Fig 3.2, for n design positions

$$\theta_{2i} = \theta_{2\text{-}0} + \theta_{2\text{-}0i} \qquad ( i = 1 \text{ to } n ) \qquad\qquad [\,3.2.1\,]$$

$$\text{and} \qquad \theta_{4i} = \theta_{4\text{-}0} + \theta_{4\text{-}0i} \qquad ( i = 1 \text{ to } n ). \qquad\qquad [\,3.2.2\,]$$

Let $\phi_{4i}$ be the actual angular displacement of the output link of the synthesized mechanism corresponding to the input angle $\theta_{2i}$. Writing the well known Freudenstein's equation [1] for the $i^{th}$ position, we get

$$D_1 \cos\theta_{2i} - D_2 \cos\phi_{4i} + D_3 = \cos( \theta_{2i} - \phi_{4i} ), \qquad\qquad [\,3.2.3\,]$$

where

$$D_1 = L_1 / L_4, \quad D_2 = L_1 / L_2$$

and

$$D_3 = ( L_1^2 + L_2^2 - L_3^2 + L_4^2 ) / ( 2 L_2 L_4 ). \qquad\qquad [\,3.2.4\,]$$

If the angle $\phi_{4i}$ is replaced by $\theta_{4i}$ ( which is available at $\theta_{2i}$ ) then Equation 3.2.3 is not satisfied in general and the difference between the L.H.S. and the R.H.S. of Equation 3.2.3 gives a small error. While using the *least square technique*, let e be the error defined as

$$e = \sum_{i=1}^{n} [ D_1 \cos\theta_{2i} - D_2 \cos\theta_{4i} + D_3 - \cos( \theta_{2i} - \theta_{4i} ) ]^2 \qquad [\,3.2.5\,]$$

Now, for the error e to be minimum, we take the partial derivatives of e with respect to $D_1$, $D_2$ and $D_3$ separately, and set each of the derivatives to zero. Thus

$$\frac{\partial e}{\partial D_1} = 2 \sum_{i=1}^{n} [ D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos ( \theta_{2i} - \theta_{4i} ) ] \cos \theta_{2i}$$

$$= 0, \qquad [ 3.2.6 ]$$

$$\frac{\partial e}{\partial D_2} = -2 \sum_{i=1}^{n} [ D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos ( \theta_{2i} - \theta_{4i} ) ] \cos \theta_{4i}$$

$$= 0 \qquad [ 3.2.7 ]$$

and $$\frac{\partial e}{\partial D_3} = 2 \sum_{i=1}^{n} [ D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos ( \theta_{2i} - \theta_{4i} ) ]$$

$$= 0. \qquad [ 3.2.8 ]$$

The above equations are rearranged to obtain

$$D_1 \sum_{i=1}^{n} \cos^2 \theta_{2i} - D_2 \sum_{i=1}^{n} \cos \theta_{4i} \cos \theta_{2i} + D_3 \sum_{i=1}^{n} \cos \theta_{2i}$$

$$= \sum_{i=1}^{n} \cos ( \theta_{2i} - \theta_{4i} ) \cos \theta_{2i}, \qquad [ 3.2.9 ]$$

$$D_1 \sum_{i=1}^{n} \cos \theta_{2i} \cos \theta_{4i} - D_2 \sum_{i=1}^{n} \cos^2 \theta_{4i} + D_3 \sum_{i=1}^{n} \cos \theta_{4i}$$

$$= \sum_{i=1}^{n} \cos ( \theta_{2i} - \theta_{4i} ) \cos \theta_{4i} \qquad [ 3.2.10 ]$$

and $$D_1 \sum_{i=1}^{n} \cos \theta_{2i} - D_2 \sum_{i=1}^{n} \cos \theta_{4i} + D_3 \sum_{i=1}^{n} 1$$

$$= \sum_{i=1}^{n} \cos ( \theta_{2i} - \theta_{4i} ) \qquad [ 3.2.11 ]$$

The above three equations are linear, simultaneous and non homogeneous in three unknowns, $D_1$, $D_2$ and $D_3$. Hence, these can be solved by using the Cramer's rule.

Finally from Equation 3.2.4 we get the link lengths. By changing the assumed values of $\theta_{2\text{-}0}$ and $\theta_{4\text{-}0}$, we can reduce the absolute value of ( $\theta_{4i} - \phi_{4i}$ ) at each design position and thus get a better solution.

**Flow chart with an illustration** - The programming for the optimized function generation problem can be easily understood from Chart 3.2. There are mainly two variables

( $\theta_{2\text{-}0}$ and $\theta_{4\text{-}0}$ ) to be assumed and one more parameter $L_1$ is used to define a unique mechanism. An induced condition ( Sqrt3 $\geq$ 0 ), mentioned in the Chart 3.2, can easily be satisfied. For obtaining a better solution, one should minimize $| \theta_{4\text{-}0} - \phi_{4i} |$ by changing the initially assumed values. After computing the link lengths, DRAW_4R( ) is executed to animate the mechanism. For verification of the result we print $| \theta_{4i} - \phi_{4i} |$ at each $\theta_{2i}$ and one can also display the result graphically.

We have considered a problem for which a set of 8 coordinated input-output angles are given. These are given below. The angles are expressed in degrees.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\theta_{2\text{-}0i}$ | 15 | 60 | 105 | 150 | 195 | 240 | 285 | 330 |
| $\theta_{4\text{-}0i}$ | 30 | 50 | 70 | 90 | 110 | 100 | 85 | 75 |

Assuming $\theta_{2\text{-}0} = 0.0$, $\theta_{4\text{-}0} = 0.0$ and $L_1 = 5$, the dimensions of the synthesized mechanism along with the plot of $\theta_2$ vs. $\phi_4$ are shown in Plot 3.4.

The errors ( i.e., $| \theta_{4i} - \phi_{4i} |$ ) at different positions are given mentioned below.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\theta_{2\text{-}0i}$ | 15 | 60 | 105 | 150 | 195 | 240 | 285 | 330 |
| $| \theta_{4i} - \phi_{4i} |$ | 1 | 2 | 0 | 0 | 6 | 6 | 10 | 12 |
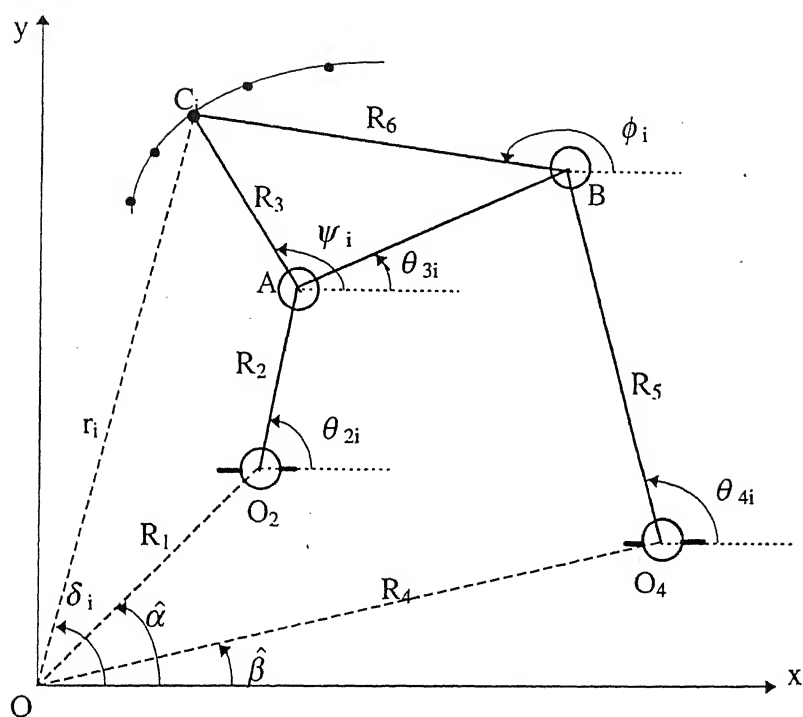
[ Fig 3.1 ]
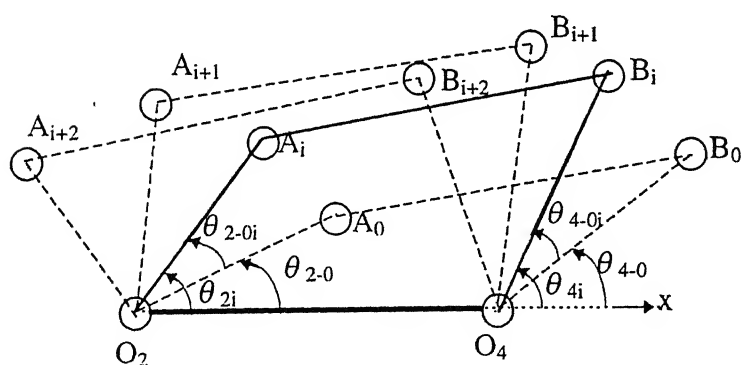


[ Fig 3.2 ]

**[ OPTIMIZATION APPROACH FOR PATH GENERATION ]**

INPUTS n,{ $(r_i, \delta_i)$ or $(x_i, y_i)$ }, $\theta_{2i}$, i=1, n ;

Assume $\hat{\alpha}$ $\hat{\beta}$ $\phi_1$

Select -
solno = ?, (1,2,3,4)
Lmaxmin = ?
Crank-rocker (Y/N) ?

**80**

Compute $l_1, l_2, l_3$ from Eqns 3.1.22a, b, c
$m_1, m_2, m_3$ from Eqns 3.1.23a, b, c

Change $\hat{\alpha}$

No ← Quad1* $\geq 0$

**10**

Yes

Compute $\lambda[1], \lambda[2]$ from 3.1.14

Set q=0

DO **20**
k = 1,2

Compute $R_1[k], R_2[k]$ from Eqn 3.1.7

**15**

45

Sqrt2* ≥ 0

No

Change ( ? )

$\hat{\alpha}$ → 10

$\hat{\beta}$

$\phi_1$

60    30

Yes

Compute R₅[q] from Eqn 3.1.7

Yes

40

p < 2

No

Yes

20

k < 2

No

Compute
L₁ [solno],
L₂ [solno],
L₃ [solno],
L₄ [solno],
R [solno] & γ [solno],
From Equation 3.1.27

30

No

Is A
Crank-rocker
?

Yes

Is Crank-rocker
Required ( Y/N )

Yes

No

65

[ Chart 3.1 ]

# [ OPTIMIZATION APPROACH FOR FUNCTION GENERATION ]

INPUTS n, $\theta_{2-0i}$, $\theta_{4-0i}$

Assume $\theta_{2-0}$, $\theta_{4-0}$, $L_1$

(10)

Compute $D_1$, $D_2$ & $D_3$ from
Eqns 3.2.9, 3.2.10 & 3.2.11

$L_2 = L_1 / D_2$
$L_4 = L_1 / D_1$

Sqrt3* $\geq$ 0 — No → (10)

$*Sqrt3 = (L_1^2 + L_2^2 + L_4^2 - 2L_2L_4D_3)$

Yes

$L_3 = \sqrt{\ (Sqrt3)}$

Compute $\phi_{4-0i}$
Print $|\phi_{4-0i} - \theta_{4-0i}|$

Is Errors
Acceptable — No → (10)

Yes

DRAW_4R( )

[ Chart 3.2 ]

L1 = 2.0
L2 = 1.0
L3 = 2.0
L4 = 1.0
R = 3.0
Gama = 120 degree
A Grashofian
A Crank-rocker

Lmax / Lmin = 2.0

Min transmission
Angle = 28.96 degree

[ Plot 3.1 ]

L1 = 1.7329
L2 = 1.0
L3 = 5.3659
L4 = 4.7002
R = 3.0
Gama = 72.93 degree
A Grashofian
A Crank-rocker

Lmax / Lmin = 5.366

Min transmission
Angle = 3.496 degree

After weighting the
error function

[ Plot 3.2 ]

L1 = 2.0219
L2 = 1.0
L3 = 6.0742
L4 = 5.143
R = 3.0
Gama = 77.10 degree
A Grashofian
A Crank-rocker

Lmax / Lmin = 6.074

Min transmission
Angle = 4.256 degree

After weighting the
error function

[ Plot 3.3 ]

L1 = 5.0
L2 = 1.734
L3 = 7.4585
L4 = 4.5599
R = 3.0
Gama = 77.10 degree
A Grashofian
A Crank-rocker

Lmax / Lmin = 4.301

Min transmission
Angle = 14.83 degree

[ Plot 3.4 ]

# 4. COUPLER CURVES

In this chapter, two important aspects of 4R coupler curves are presented. The first one is related to the generation of two cusps by a coupler point of a 4R mechanism. The second one is concerned with the coupler curve cognate linkages.

## 4.1 TWO CUSPS PROBLEM

The objective is to design a crank-rocker mechanism, in which a coupler point has two cusps at prescribed locations and the amount of crank rotation as the coupler point moves from one cusp to the another is also prescribed.
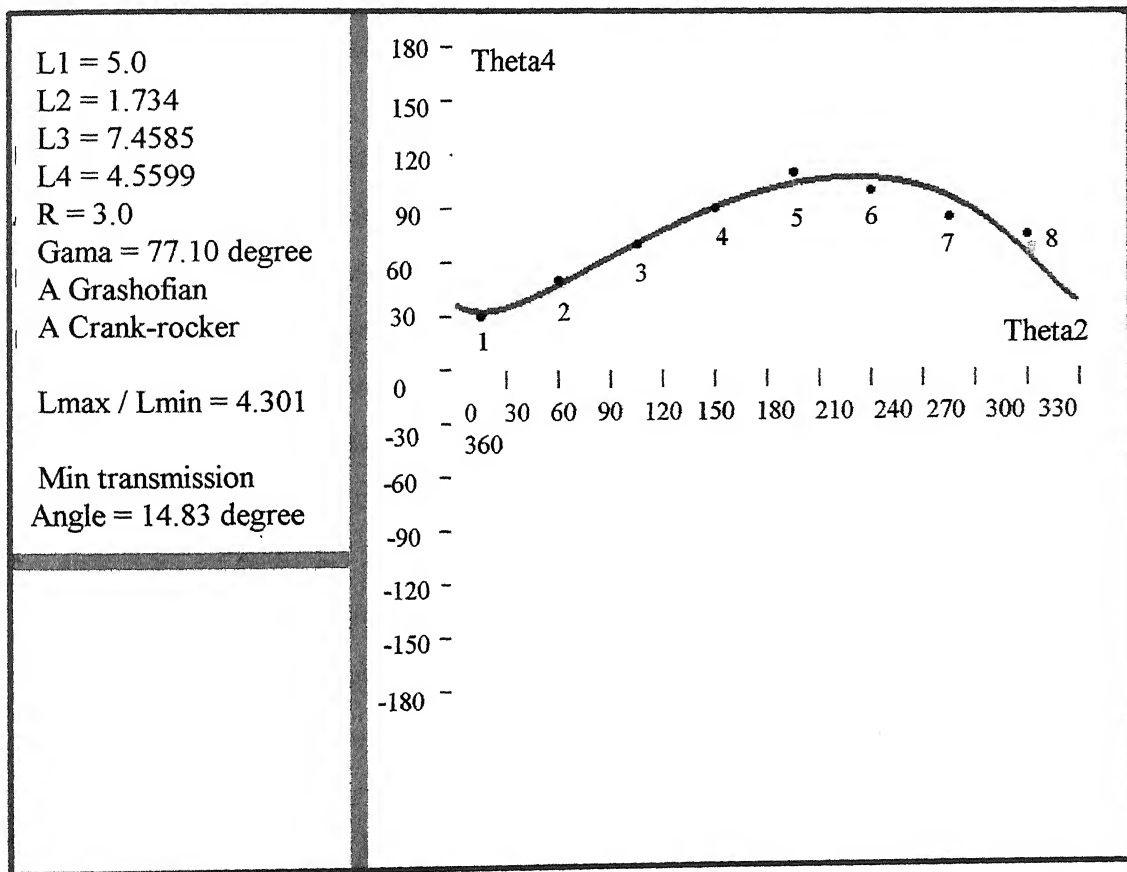
Before solving this problem, we should note the following result[1].

For every crank-rocker mechanism, there exists a coupler point which has two cusps. This coupler point can be located at the instantaneous center corresponding to the configurations when the crank-pin A lies on the *Thales circle* $CR_T$ ( a circle with $O_2O_4$ as a diameter ) ( Fig 4.1 ).

It can be easily proved that the sum of the crank rotation ( $\theta_{2c}$ ) and the rocker rotation ( $\theta_{4c}$ ) as the coupler point goes from one cusp to the another is $\pi$. This can be proved from Fig 4.1 as explained below.

Since $\triangle$ $O_4B_2A_2$ is obtained by rotating $\triangle$ $O_4B_1A_1$ about $O_4$ through an angle $\theta_{4c}$ and $\angle$ $B_2O_4B_1 = \angle$ $C_2O_4C_1 = \theta_{4c}$, we get $\angle$ $A_2O_4A_1 = \theta_{4c}$.

But $A_1$, $A_2$, $O_2$ and $O_4$ are on the same circle $CR_T$, therefore

$$\theta_{2c} + \theta_{4c} = \pi \qquad \text{or} \qquad \theta_{4c} = \pi - \theta_{2c}. \qquad\qquad [\,4.1.1\,]$$

Let $C_1 \equiv (x_1, y_1)$ and $C_2 \equiv (x_2, y_2)$ are the prescribed locations of the cusps whereas $\theta_{2c}$ is the given crank rotation as the coupler point goes from one cusp to the another.

It is also known, that the double point ( if it exists on the coupler curve ) lies on a circle

known as *circle of singular foci*[1]. But, the circle of singular foci passes through the fixed hinges ( i.e. $O_2$ and $O_4$ ). Hence we can say that $C_1$, $C_2$, $O_2$ and $O_4$ will lie on the same circle.

From Fig 4.1, concentrating on $\Delta$'s $O_4A_1C_1$ and $O_4A_2C_2$, we observe that

$$\angle A_1O_4C_1 \;=\; \angle A_1O_4B_2 + \theta_{4c} \;=\; \angle A_2O_4C_2,$$

$$\angle C_1A_1O_4 \;=\; \angle C_2A_2O_4 \;=\; \pi / 2$$

and

$$O_4A_1 \;=\; O_4A_2.$$

Therefore, we can say that $\Delta$ $O_4A_1C_1 \equiv \Delta$ $O_4A_2C_2$ and $O_4C_1 = O_4C_2$.

Thus, for the synthesis of the desired mechanism, we draw a circle ( $CR_1$ ) with the following characteristics :

[i] It passes through $C_1$ and $C_2$.

[ii] The angle subtended by $C_1C_2$ at $O_4 = ( \pi - \theta_{2c} )$.

[iii] $O_4C_1 = O_4C_2$.

In other words, we can draw a circle $CR_1$ passing through $C_1$, $C_2$ and $O_4$ such that $O_4$ is on the mid-normal of $C_1C_2$ and the angle subtended by $C_1C_2$ at $O_4$ is ( $\pi - \theta_{2c}$ ) ( Fig 4.2 ).

Suppose

$C_0 \equiv ( x_{C0}, y_{C0} )$ is the center of $CR_1$ ( of radius $r_1$ ),

$O_4 \equiv ( x_{O4}, y_{O4} )$,

$p$ = the distance of $C_0$ from $C_1C_2$

and $C_{12}$ = distance between $C_1$ and $C_2$.

Therefore, from Fig 4.2

$$C_{12} = \sqrt{\;} [ ( x_1 - x_2 )^2 + ( y_1 - y_2 )^2 ], \qquad\qquad [\,4.1.2\,]$$

$$\delta \;( \text{slope of } C_1C_2 ) = \tan^{-1} [ ( y_1 - y_2 ) / ( x_1 - x_2 ) ], \qquad [\,4.1.3\,]$$

$$r_1 = C_{12} / ( 2 \sin \theta_{2c} ), \qquad\qquad [\,4.1.4\,]$$

$$p = \sqrt{[\, r_1^2 - ( C_{12}^2 / 4 )\,]}, \qquad\qquad [\,4.1.5\,]$$

$$x_{C0} = x_2 + ( C_{12} / 2 ) \cos \delta + p \sin \delta, \qquad\qquad [\,4.1.6a\,]$$

$$y_{C0} = y_2 + ( C_{12} / 2 ) \sin \delta - p \cos \delta, \qquad\qquad [\,4.1.6b\,]$$

$$x_{O4} = x_2 + ( C_{12} / 2 ) \cos \delta + ( p + r_1 ) \sin \delta \qquad\qquad [\,4.1.7a\,]$$

and

$$y_{O4} = y_2 + ( C_{12} / 2 ) \sin \delta - ( p + r_1 ) \cos \delta. \qquad\qquad [\,4.1.7b\,]$$

Writing the equation of the circle $CR_1$, we have

$$( x - x_{C0} )^2 + ( y - y_{C0} )^2 = r_1^2. \qquad\qquad [\,4.1.8\,]$$

Since $O_2$ [ $\equiv$ ( $x_{O2}$, $y_{O2}$ ) ] also lies on the same circle $CR_1$, $x_{O2}$ can be chosen anywhere on the circle between ( $x_{C0} - r_1$ ) and ( $x_{C0} + r_1$ ). Hence, from Equation 4.1.8 $y_{O2}$ is known. Thus $O_2$ and $O_4$ are known, and we can draw the *Thales circle* $CR_T$.

Let $C_T$ [ $\equiv$ ( $x_{CT}$, $y_{CT}$ ) ] and $r_t$ be the center and radius of the circle $CR_T$, respectively.

Therefore, $\qquad x_{CT} = ( x_{O2} + x_{O4} ) / 2; \qquad\qquad [\,4.1.9a\,]$

$$y_{CT} = ( y_{O2} + y_{O4} ) / 2 \qquad\qquad [\,4.1.9b\,]$$

and

$$r_t = [\, \sqrt{\{ ( x_{O4} - x_{O2} )^2 + ( y_{O4} - y_{O2} )^2 \}}\,] / 2. \qquad\qquad [\,4.1.10\,]$$

Equation of the circle $CR_T$ is given by

$$( x - x_{CT} )^2 + ( y - y_{CT} )^2 = r_t^2. \qquad [ 4.1.11 ]$$

The equation of the line $O_2C_1$ is

$$y = m x + c, \qquad [ 4.1.12 ]$$

where

$$m = ( y_1 - y_{O2} ) / ( x_1 - x_{O2} ) \qquad [ 4.1.13a ]$$

and

$$c = y_{O2} - m x_{O2} \qquad [ 4.1.13b ]$$

We know that the crank-pin lies on the point of intersection of the crank circle and the Thales circle when the coupler point lies at either of the cusps. A cusp is an instantaneous center of velocity of the coupler. Also for a 4R mechanism, the instantaneous center of the coupler relative to the fixed link $O_2O_4$ lies at the intersection of lines drawn through the input and the output links ( using the well-known *theorem of three centers* ). In other words, at the configuration when the coupler point lies at the instantaneous center ( or cusp on the coupler curve ), the crank-pin lies on the line joining the fixed end of the crank and the instantaneous center ( or cusp on the coupler curve). On this basis, we can say that the crank-pins $A_1$ [ say $(x_{A1}, y_{A1})$ ] and $A_2$ [ say $(x_{A2}, y_{A2})$ ] ( lying on the Thales circle $CR_T$ ) lie on the lines $O_2C_1$ and $O_2C_2$ respectively ( Fig 4.2 ).

Now, from Equations 4.1.11 and 4.1.12

$$( x - x_{CT} )^2 + (m x + c - y_{CT} )^2 = r_t^2$$

or

$$( 1 + m^2 ) x^2 + 2 ( t_1 + m t_2 ) x + ( t_1^2 + t_2^2 - r_t^2 ) = 0, \qquad [ 4.1.14 ]$$

where

$$t_1 = - x_{CT} \qquad \text{and} \qquad t_2 = c - y_{CT}.$$

The quadratic Equation 4.1.14 gives two values of x, as the line $O_2C_1$ cuts the circle $CR_T$ at two points. Solving this equation, we get the x coordinate of $A_1$ i.e. $x_{A1}$. Then, from

Equations 4.1.12, 4.1.13a and 4.1.13b we can find the y _ coordinate of $A_1$ i.e. $y_{A1}$. Hence $A_1$ is known.

To find all the link lengths, we have to locate the point $B_1$ [ say ( $x_{B1}$, $y_{B1}$ ) ]. Since the point $B_1$ ( the free end of the follower when the coupler point lies at the cusp $C_1$ ) lies anywhere on the line $O_4C_1$, Therefore, we can chose $x_{B1}$ between $x_1$ ( the x _ coordinate of $C_1$ ) and $x_{O4}$ (the x _ coordinate of $O_4$ ).

Now the equation of line $O_4C_1$ is written as

$$y = m_1 x + c_1,$$   [ 4.1.15 ]

where

$$m_1 = ( y_1 - y_{O4} ) / ( x_1 - x_{O4} )$$   [ 4.1.16a ]

and

$$c_1 = y_{O4} - m_1 x_{O4}.$$   [ 4.1.16b ]


Therefore, we can find $y_{B1}$ corresponding to $x_{B1}$ on the line $O_4C_1$ ( from Equation 4.1.15 ). Depending upon the selection of $x_{O2}$ and $x_{B1}$ we can find many solutions but we control these selections to get a crank-rocker mechanism with $O_2A_1$ as the shortest link. Thus a crank-rocker mechanism $O_2A_1B_1O_4$ is designed.


**Flow chart with an illustration** - Chart 4.1 clearly explains the steps used to design a crank-rocker mechanism for the present problem. The assumptions made are


$$x_{O2} = x_{C0} - r_1/2$$

and   $$x_{B1} = (x_1 + x_{O4})/2,$$


which can be changed to get a required mechanism. As an illustration, we have selected two cusps at

$$C_1 \equiv ( 10, 8 ),$$

$$C_2 \equiv ( 1, 1 )$$

and   $$\theta_{2c} = 120^0.$$

The designed mechanism with coupler curve and link lengths are shown in Plot 4.1.

## 4.2 COGNATE LINKAGES

The term *cognate* ( coined by Hartenberg and Denavit ) of a mechanism means a different mechanism which generates the same coupler curve. The most important theorem[1], developed by Samuel Roberts ( 1875 ) and Chebyschev ( 1878 ) indepen-dently, states that *three different planar 4R linkages trace identical coupler curves.*

Let $O_2ABO_4$ is a given crank-rocker mechanism and C is its coupler point ( Fig 4.3 ). There exist two 4R cognate linkages of this mechanism. To get the cognate linkages, first draw parallelograms $O_2ACD$ and $O_4BCG$. On sides DC and CG, make triangles EDC and FCG respectively similar to the triangle CAB. Again draw a parallelogram OECF and make the hinge O fixed. This ten-link mechanism is found[1] constrained ( with degree of freedom = 1 ). Thus, the common coupler point C of mechanisms $O_2ABO_4$, $O_2DEO$ and $OFGO_4$ generates the same coupler curve. The mechanism $O_2DEO$ is called the *left-hand cognate* and $OFGO_4$ the *right-hand cognate.*

It should be noted [1] that, $\Delta\ OO_2O_4$ is also similar to $\Delta\ CAB$.

Now, suppose

$$O_2O_4 = L_1, \ O_2A = L_2, \ AB = L_3,$$

$$O_4B = L_4, \ AC = R \text{ and } BC = S.$$

The unknown design parameters of both cognates can be found analytically in the following manner.

[i] For $OFCGO_4$ ( Right-hand cognate ) - The design parameters $RG\_L_1$, $RG\_L_2$, $RG\_L_3$, $RG\_L_4$, $RG\_R$ and $RG\_S$ are given as below.

From Fig 4.3,

$$RG\_L_1 = OO_4 = (\ O_2O_4\ )\ (\ BC\ /\ AB\ )$$

$$(\text{ from similar } \Delta\text{ 's } OO_2O_4 \text{ and } CAB\ ),$$

or $\quad RG\_L_1 = L_1\ (\ S\ /\ L_3\ ),$ $\qquad\qquad$ [ 4.2.1 ]

$$RG\_L_2 = OF = EC = LG\_S = (\ S\ /\ L3\ )\ L_2,\qquad\qquad [\ 4.2.2\ ]$$

$$RG\_L_3 = FG = (\sin \gamma_1 / \sin \gamma_3) GC \qquad (\text{from } \Delta \text{ CDE})$$

or $\qquad RG\_L_3 = (S / L3) L_4 \qquad (\text{from } \Delta \text{ ABC}),$

$$[4.2.3]$$

$$RG\_L_4 = O_4G = S, \qquad [4.2.4]$$

$$RG\_R = FC = (\sin \gamma_2 / \sin \gamma_3) GC \qquad (\text{from } \Delta \text{ CDE}),$$

or $\qquad RG\_R = (R / L3) L_4 \qquad (\text{from } \Delta \text{ ABC}),$

$$[4.2.5]$$

and

$$RG\_S = GC = L_4. \qquad [4.2.6]$$

[ii] For $O_2DCEO$ ( Left-hand cognate ) - The design parameters $LG\_L_1$, $LG\_L_2$, $LG\_L_3$, $LG\_L_4$, $LG\_R$ and $LG\_S$ are given as below.

From Fig 4.3,

$$LG\_L_1 = O_2O = (O_2O_4)(AC / AB)$$

$$(\text{from similar } \Delta\text{'s } OO_2O_4 \text{ and CAB}),$$

or $\qquad LG\_L_1 = L_1 (R / L_3), \qquad [4.2.7]$

$$LG\_L_2 = O_2D = AC = R, \qquad [4.2.8]$$

$$LG\_L_3 = DE = (\sin \gamma_2 / \sin \gamma_3) DC \qquad (\text{from } \Delta \text{ CDE})$$

or $\qquad LG\_L_3 = (R / L3) L_2 \qquad (\text{from } \Delta \text{ ABC}),$

$$[4.2.9]$$

$$LG\_L_4 = OE = FC = RG\_R = (R / L3) L_4, \qquad [4.2.10]$$

$$LG\_R = DC = O_2A = L_2 \qquad [4.2.11]$$

and

$$LG\_S = EC = ( \sin \gamma_1 / \sin \gamma_3 ) \, DC \qquad ( \text{from } \Delta \text{ CDE} )$$

or $\qquad LG\_S = ( S / L3 ) \, L_2 \qquad\qquad ( \text{from } \Delta \text{ ABC} ).$

[ 4.2.12 ]

One can easily prove that the maximum-minimum link length ratio ( $L_{max} / L_{min}$ ) for the original and its cognate linkages are same.

We can analyze the angular relationships of the various links of different cognate linkages in the following manner.

Observing the parallelograms $O_2ACD$, $O_4BCG$ and $OECF$ we can say that

| Original Linkage | Left - Hand Cognate | Right - Hand Cognate |
|---|---|---|
| Angular motion of link **2** $\equiv$ | Angular motion of link **6** $\equiv$ | Angular motion of link **8** |
| Angular motion of link **3** $\equiv$ | Angular motion of link **5** $\equiv$ | Angular motion of link **10** |
| Angular motion of link **4** $\equiv$ | Angular motion of link **7** $\equiv$ | Angular motion of link **9**. |

Considering the above relationships we can conclude that, if the original linkage ( $O_2ABO_4$ ) is a crank-rocker with say 2, as the crank ( when 3 and 4 must oscillate ), then the right-hand cognate is also a crank-rocker, with 8 as the crank ; however, the left-hand cognate then must be a double-rocker ( with the coupler 6 undergoing the full rotation ). Similarly both the cognates of a double-rocker linkage ( say $O_2DCEO$ ) are crank-rockers ( $OFCGO_4$ and $O_2ACBO_4$ ).

It should be noted that the transmission angle ( $\mu_{min}$ ) for crank-rocker cognate linkages are same.

From Equation 2.2.1, the transmission angle for the mechanism is

$$\mu = \cos^{-1} [ \{ ( L_3^2 + L_4^2 ) - ( L_1^2 + L_2^2 ) - 2 \, L_1 \, L_2 \cos \theta_2 \} / ( 2 \, L_3 \, L_4 ) ].$$

[ 4.2.13 ]

The transmission angle for the right-hand cognate is given by ( replacing L's by RG_L's )

$$\mu_{right} = \cos^{-1} \left[ \left\{ \left( RG\_L_3^2 + RG\_L_4^2 \right) - \left( RG\_L_1^2 + RG\_L_2^2 \right) \right. \right.$$
$$\left. \left. - 2\ RG\_L_1\ RG\_L_2\ \cos\theta_{2r} \right\} / \left( 2\ RG\_L_3\ RG\_L_4 \right) \right]$$

where $\theta_{2r}$ represents the rotation of link 8.

Substituting the values of $RG\_L_1$, $RG\_L_2$, $RG\_L_3$ and $RG\_L_4$ in the above equation ( from Equations 4.2.1 - 4.2.4 ), we get

$$\mu_{right} = \cos^{-1} \left[ \left[ \left\{ \left( L_4\ S/L_3 \right)^2 + S^2 \right\} - \left\{ \left( L_1\ S/L_3 \right)^2 + \left( L_2\ S/L_3 \right)^2 \right\} \right. \right.$$
$$\left. \left. - 2\ \left( L_1\ S/L_3 \right) \left( L_2\ S/L_3 \right)\ \cos\theta_{2r} \right] / \left\{ \left( L_4\ S/L_3 \right)\ S \right\} \right]$$

or

$$\mu_{right} = \cos^{-1} \left[ \left\{ \left( L_3^2 + L_4^2 \right) - \left( L_1^2 + L_2^2 \right) - 2\ L_1\ L_2\ \cos\theta_{2r} \right\} / \left( 2\ L_3\ L_4 \right) \right].$$
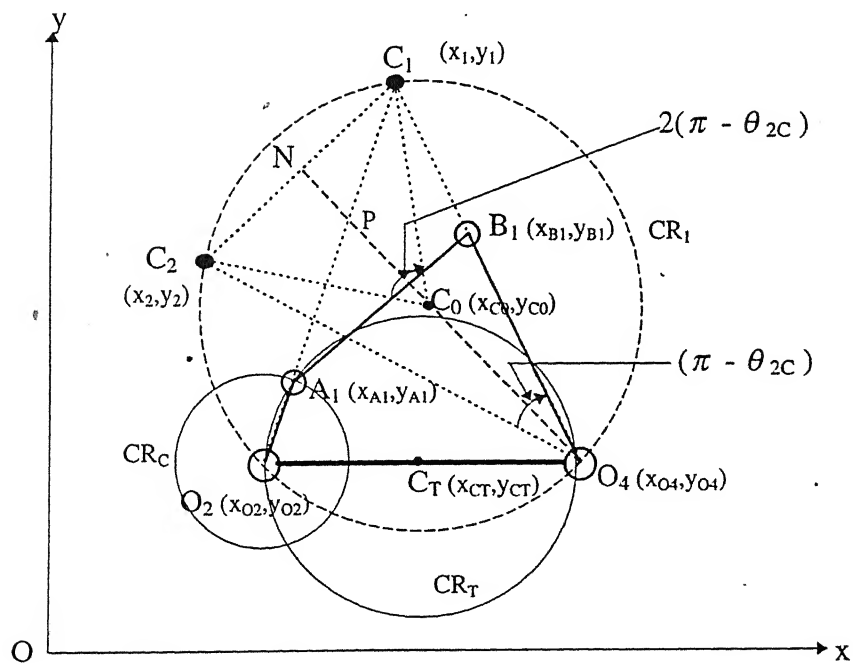
[ 4.2.14 ]

Suppose the original mechanism ( $O_2ABO_4$ ) is a crank-rocker. Since, link 8 is equivalent to link 2 regarding the angular motion, we can say that $\theta_{2r}$ and $\theta_2$ posses the same values for calculation of the minimum transmission angle. Hence, from Equations 4.2.13 and 4.2.14, $\mu_{right}$ and $\mu$ are same.

**Illustration -** For every crank-rocker and Grashofian double-rocker mechanisms, we can visualize the other two cognates using the present software. While solving a path generation problem for three precision points, we get a double-rocker mechanism, but as per the requirement of a crank-rocker mechanism one can select one of its cognates. Plot 4.2 shows the original mechanism ( double-rocker ) and its cognate linkages ( crank-rockers ) along with the coupler curve passing through the specified points in a path generation problem.
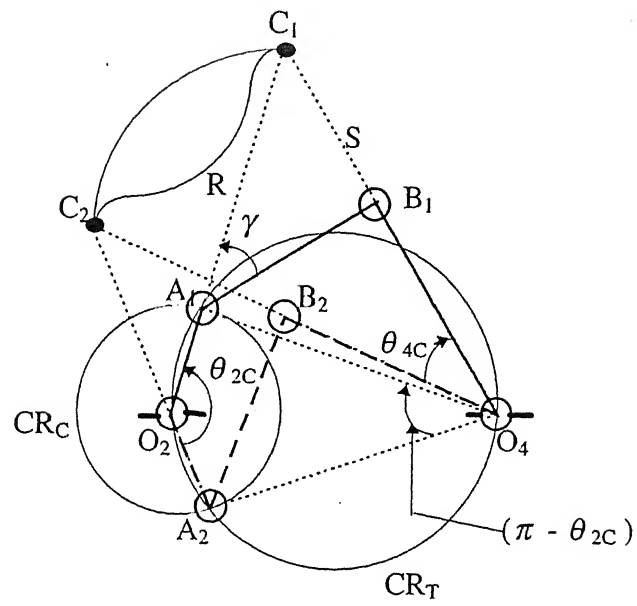
[ Fig 4.1 ]



[ Fig 4.2 ]

[ Fig 4.3 ]

[ TWO CUSPS PROBLEMS ]

INPUTS $C_1(x_1,y_1)$, $C_2(x_2,y_2)$, $\theta_{2c}$

Compute
$\theta_{4c}$ $C_{12}$, $\delta$, $r_1$, p, $x_{C0}$, $y_{C0}$,
$x_{O4}$, $y_{O4}$ ;
from Eqns 4.1.1 - 4.1.7

*Assume $x_{O2}$

Compute $y_{O2}$ from Eqn. 4.1.8,
$(x_{CT}, y_{CT}, r_t)$ from Eqns. 4.1.9-10
& (m,c) from Equation 4.1.13

Change $x_{O2}$ ← No — Sqrt4 $\geq$ 0

$Sqrt4 = [(t_1 + m\,t_2)^2 - (1+m^2)$
$\qquad\qquad\qquad (t_1^2 + t_2^2 - r_t^2)]$
where $t_1 = -x_{CT}$, $t_2 = c - y_{CT}$

Yes

Compute $x_{A1}$ from Eqn. 4.1.14,
$y_{A1}$ from Eqns. 4.1.12-13

** Assume $x_{B1}$ between
$x_1$ and $x_{O4}$

No

Compute $y_{B1}$ from
Eqns. 4.1.15 - 16
& Link Lengths -
$L_1 = O_2O_4$, $L_2 = O_2A_1$
$L_3 = A_1B_1$, $L_4 = O_4B_1$

Is Crank-rocker

Yes

DRAW_4R( )

Initially Assumed values are
* $x_{O2} = x_{C0} - r_1/2$
** $x_{B1} = (x_1 + x_{O4})/2$

[ Chart 4.1 ]

L1 = 7.3502
L2 = 3.6751
L3 = 5.7009
L4 = 5.7009
R = 9.4594
Gama = 33.94 degree
A Grashofian
A Crank-rocker

Lmax / Lmin = 2.0

Min transmission
Angle = 29.53 degree

[ Plot 4.1 ]

L1 = 4.0
L2 = 3.7612
L3 = 2.531
L4 = 3.5503
R = 1.6259
Gama = 65.19 degree
A Grashofian
A Double-rocker

Lmax / Lmin = 1.58

Min transmission
Angle = 0.0 degree

|                    | Left Cognate | Original Mechanism | Right Cognate |
|--------------------|--------------|--------------------|---------------|
|                    | Crank-rocker | Double-rocker      | Crank-rocker  |
| Lmax / Lmin        | 1.58         | 1.58               | 1.58          |
| Min. trans. Angle  | 22.92        | 0.0                | 22.92         |

[ Plot 4.2 ]

# 5. KINEMATIC CONDITIONS

In this chapter, we are going to present the synthesis of 4R mechanisms for prescribed kinematic parameters ( e.g. angular velocities and angular accelerations etc. ) of some of its links. Let $\theta_i$, $\omega_i$ and $\alpha_i$ are the angular displacement, angular velocity and angular acceleration of the $i^{th}$ link as shown in Fig 5.1. Depending upon the values of i we can have two cases.

**Case 1.** If $\theta_i$, $\omega_i$ and $\alpha_i$ for input ( i = 2 ) and output ( i = 4 ) links are given.

Using Freudenstein 's equation we have

$$D_1 \cos \theta_2 + D_2 ( - \cos \theta_4 ) + D_3 = \cos ( \theta_2 - \theta_4 ), \qquad [5.1]$$

where

$$D_1 = L_1 / L_4,$$

$$D_2 = L_1 / L_2 \qquad [5.2]$$

and $\quad D_3 = ( L_1^2 + L_2^2 - L_3^2 + L_4^2 ) / ( 2 L_2 L_4 )$

with $L_i$ as the length of the $i^{th}$ link, i = 1 to 4.

Differentiating Equation 5.1 with respect to time, we get

$$D_1 ( \omega_2 \sin \theta_2 ) + D_2 ( - \omega_4 \sin \theta_4 ) = ( \omega_2 - \omega_4 ) \{ \sin ( \theta_2 - \theta_4 ) \} \qquad [5.3]$$

and differentiating Equation 5.3 once more with respect to time, we obtain

$$D_1 ( \omega_2^2 \cos \theta_2 + \alpha_2 \sin \theta_2 ) + D_2 ( - \omega_4^2 \cos \theta_4 - \alpha_4 \sin \theta_4 )$$
$$= ( \omega_2 - \omega_4 )^2 \{ \cos ( \theta_2 - \theta_4 ) \} + ( \alpha_2 - \alpha_4 ) \{ \sin ( \theta_2 - \theta_4 ) \}. \qquad [5.4]$$

Three design parameters, $D_1$, $D_2$ and $D_3$ can be solved from Equations 5.1, 5.3, and 5.4 using Cramer's rule. Then using Equation 5.2, and assuming any one of the link-lengths ( i.e., the scale of the link diagram ) we can have the other three link-lengths.

**Case 2.** If $\omega_i$ and $\alpha_i$ for input ( i = 2 ), coupler ( i = 3 ) and output ( i = 4 ) links are given.

Writing the horizontal and vertical components of the loop closure equation for the loop $O_2ABO_4O_2$, we get

$$L_2 \cos \theta_2 + L_3 \cos \theta_3 = L_1 + L_4 \cos \theta_4 \qquad [5.5]$$

and

$$L_2 \sin \theta_2 + L_3 \sin \theta_3 = L_4 \sin \theta_4. \qquad [5.6]$$

Equations 5.5 and 5.6 can be combined into a complex exponential form as given below.

$$L_2 \exp ( i \, \theta_2 ) + L_3 \exp ( i \, \theta_3 ) = L_1 + L_4 \exp ( i \, \theta_4 )$$

or

$$( L_2 / L_1 ) \exp ( i \, \theta_2 ) + ( L_3 / L_1 ) \exp ( i \, \theta_3 ) - ( L_4 / L_1 ) \exp ( i \, \theta_4 ) = 1$$

or

$$Z_2 + Z_3 - Z_4 = Z_1 \qquad [5.7]$$

where $Z_n$ [ $= (L_n / L_1) \exp( i \, \theta_n )$, n = 1, 2, 3, 4 ] is a complex variable. For the fixed link $\theta_1 = 0$.

Differentiating Equation 5.7 successively with respect to time, we get

$$\omega_2 Z_2 + \omega_3 Z_3 - \omega_4 Z_4 = 0 \qquad [5.8]$$

and

$$( \alpha_2 + i \, \omega_2{}^2 ) Z_2 + ( \alpha_3 + i \, \omega_3{}^2 ) Z_3 - ( \alpha_4 + i \, \omega_4{}^2 ) Z_4 = 0$$

or

$$V_2 Z_2 + V_3 Z_3 - V_4 Z_4 = 0 \qquad [5.9]$$

where $V_n$ [ $= ( \alpha_n + i \, \omega_n{}^2 )$, n = 2, 3, 4 ] is a complex variable.

From Equations 5.7 and 5.8, we get

$$\omega_{24} Z_2 + \omega_{34} Z_3 + \omega_4 Z_1 = 0 \qquad [5.10]$$

and similarly, from Equations 5.7 and 5.9, we obtain

$$V_{24} Z_2 + V_{34} Z_3 + V_4 Z_1 = 0 \qquad\qquad [\,5.11\,]$$

$$\text{where } \omega_{ij} = \omega_i - \omega_j \text{ and } V_{ij} = V_i - V_j.$$

Eliminating $Z_2$, from Equations 5.10 and 5.11, we have

$$Z_3 = [\,\{\, - V_4 + (\,\omega_4 /\, \omega_{24}\,)\, V_{24}\,\} \,/\, \{\, V_{34} - (\,\omega_{34} /\, \omega_{24}\,)\, V_{24}\,\}\,]\, Z_1.$$

$$[\,5.12\,]$$

Then, from Equation 5.10

$$Z_2 = -[\,(\,\omega_4 /\, \omega_{24}\,)\, Z_1 + (\,\omega_{34} /\, \omega_{24}\,)\, Z_3\,] \qquad\qquad [\,5.13\,]$$

and from Equation 5.7

$$Z_4 = -Z_1 + Z_2 + Z_3. \qquad\qquad [\,5.14\,]$$

Thus knowing $Z_2$, $Z_3$ and $Z_4$ ( as a function of $Z_1$ ), we can draw the mechanism assuming a suitable value for one of the link-lengths.

**Flow chart with illustrations** - The steps used in the programming are clear from Chart 5.1. After computing the link-lengths, one can use the same software for its kinematic analysis for verifying the results.

For illustration of case **1**, we have taken the following data's to design a mechanism.

$$(\,\theta_2 = 90^0,\ \omega_2 = 2 \text{ rad / sec} \text{ and } \alpha_2 = 0 \text{ rad / sec}^2\,),$$

$$(\,\theta_4 = 60^0,\ \omega_4 = 3 \text{ rad / sec} \text{ and } \alpha_4 = -1 \text{ rad / sec}^2\,)$$

and $\quad L_1 = 10$ unit.

The results obtained, are

$$L_2 = 26.602 \text{ unit,}$$

$$L_3 = 23.394 \text{ unit}$$

and $\quad L_4 = 13.654 \text{ unit} ;$

which form a non Grashofian mechanism. The angular displacements and velocities can be verified from Plot 5.1 and Plot 5.2, respectively.

For case **2**, we have considered a problem with

$$(\ \omega_2 = 10 \text{ rad / sec} \ \text{and} \ \alpha_2 = 40 \text{ rad / sec}^2\ ),$$

$$(\ \omega_3 = 4 \text{ rad / sec} \ \text{and} \ \alpha_3 = 60 \text{ rad / sec}^2\ ),$$

$$(\ \omega_4 = 20 \text{ rad / sec} \ \text{and} \ \alpha_2 = 0 \text{ rad / sec}^2\ )$$
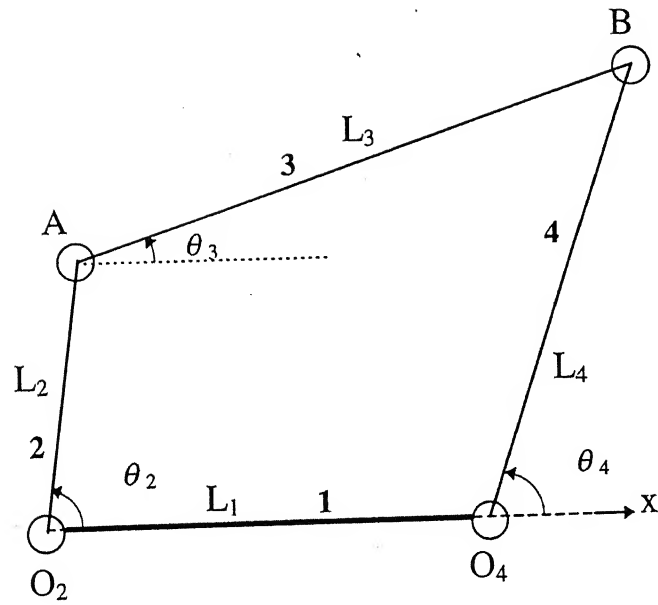
and $\quad L_1 = 10 \text{ unit.}$

The results obtained, are

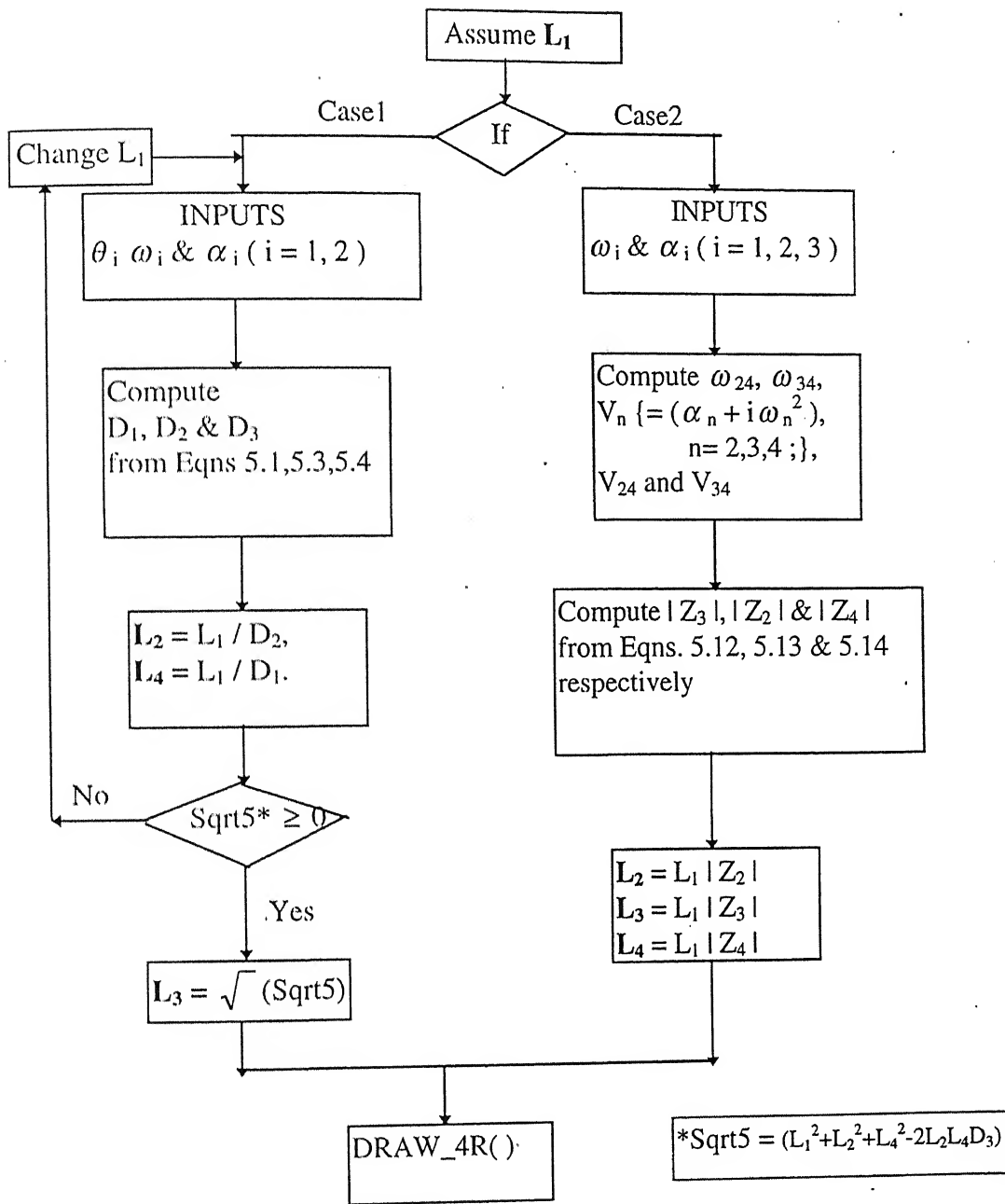$$L_2 = 18.256 \text{ unit,}$$

$$L_3 = 22.419 \text{ unit}$$

and $\quad L_4 = 5.2147 \text{ unit} ;$
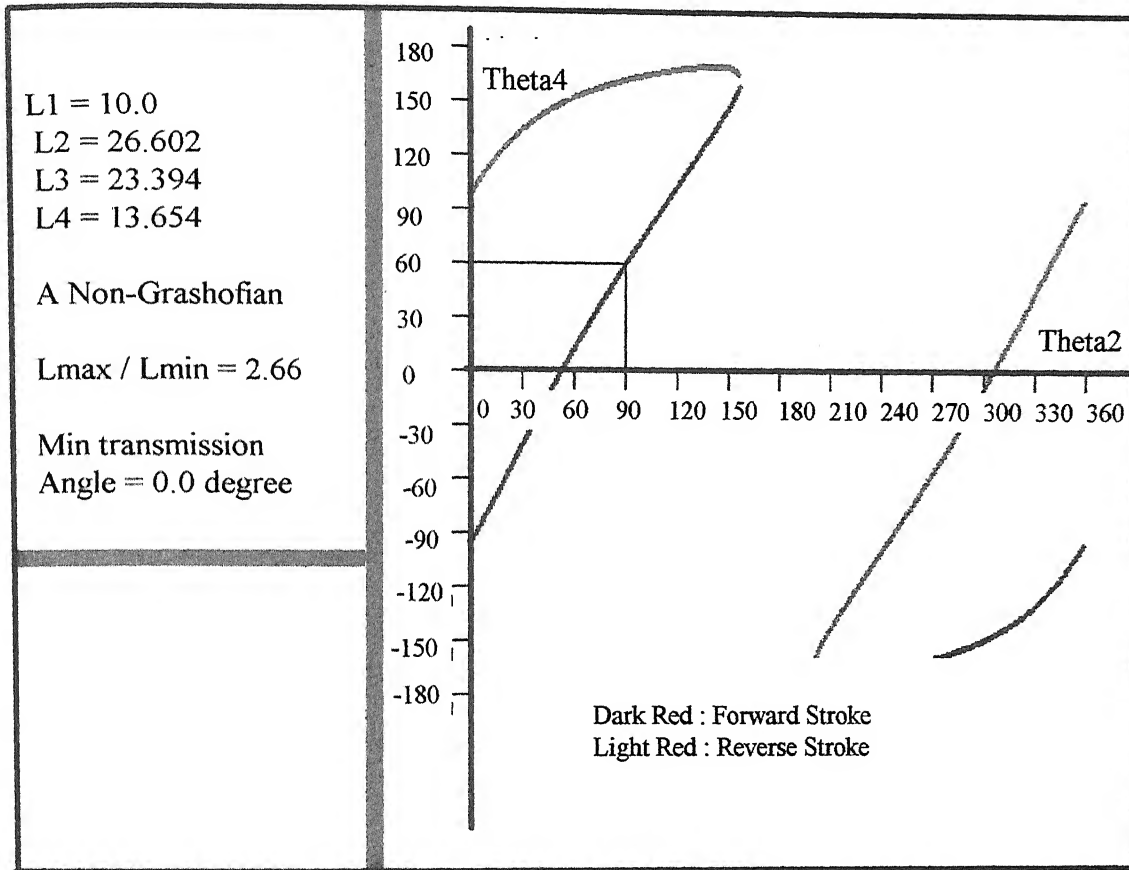
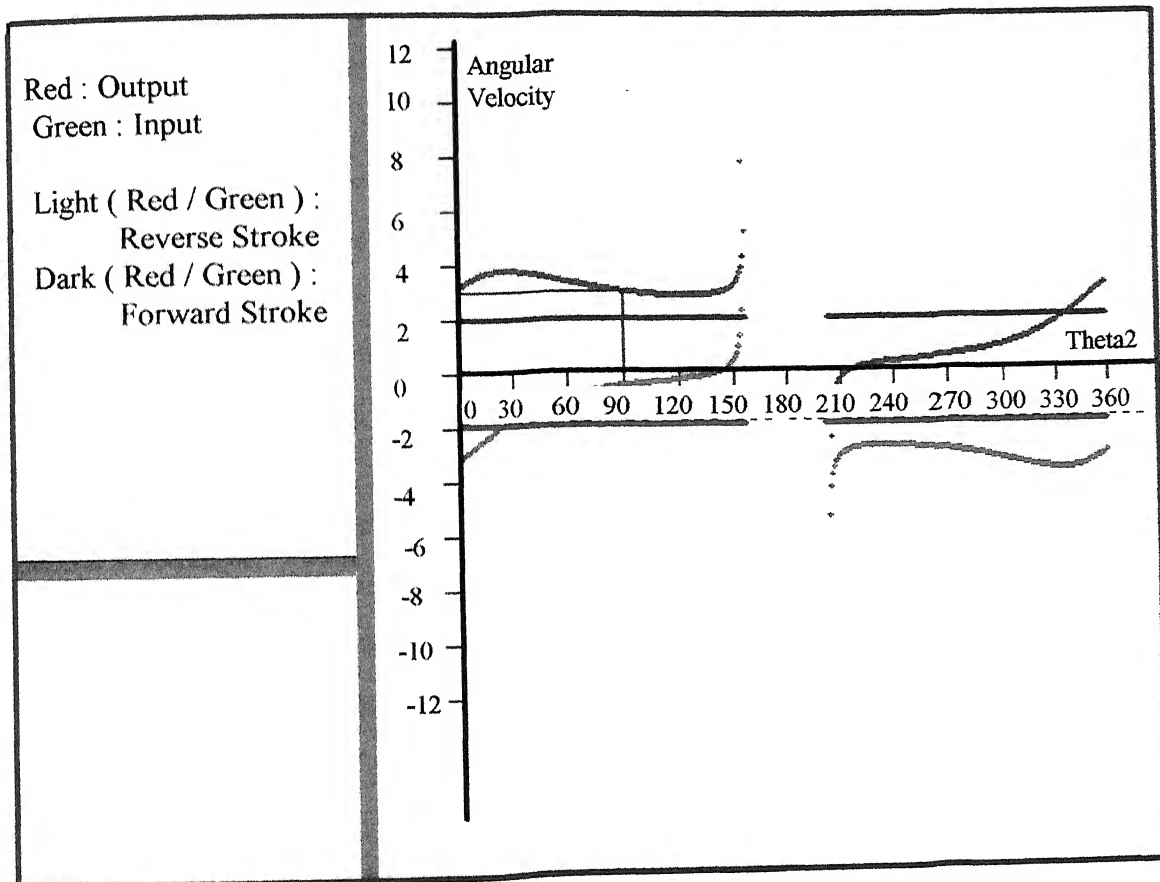which give a mechanism of the crank-rocker type.

[ Fig 5.1 ]

**[ PROBLEMS WITH KINEMATIC VARIABLES ]**



[ Chart 5.1 ]

L1 = 10.0
L2 = 26.602
L3 = 23.394
L4 = 13.654

A Non-Grashofian

Lmax / Lmin = 2.66

Min transmission
Angle = 0.0 degree

Theta4

Theta2

Dark Red : Forward Stroke
Light Red : Reverse Stroke

[ Plot 5.1 ]

Red : Output
Green : Input

Light ( Red / Green ) :
    Reverse Stroke
Dark ( Red / Green ) :
    Forward Stroke

Angular Velocity

Theta2

[ Plot 5.2 ]

# 6. CONCLUSIONS AND
# SUGGESTIONS FOR FUTURE WORK

## 6.1 CONCLUSIONS

On the basis of the results presented, the following conclusions can be drawn.

[ i ] The slider-crank mechanism for specified dead-center configurations under different cases ( discussed in section 2.1 ) can be synthesized and visualized with animation. Having all the three inputs ( H, $Q_{RR}$ and one of the three parameters E, $L_2$ and $L_3$ ), the software checks the existence of a mechanism. Also an optimum design for prescribed values H and $Q_{RR}$ can be obtained.

[ ii ] The synthesis of a crank-rocker mechanism for dead-center problems has been done without using the Volmer's nomogram. In comparison to the conventional nomogram approach, the new approach gives more accurate values of $\beta_{opt}$, ( $\mu_{min}$ )$_{max}$ and hence the link-lengths for better transmission quality. The accuracy can also be controlled at the cost of execution load of the program as number of iterations will increase.

[ iii ] The overall optimization approach for the path generation problems can design 4R mechanisms for as many specified points as one wants, but the computational load will be increased. The present software also takes care of some constraints like the maximum-minimum link-lengths ratio and the compulsion for the designed mechanism to be a crank-rocker.

In the present software, an option to weight the error function ( used for the least square technique ) according to the errors has been provided. In this way the error can be further reduced. If there is any critical point through which the coupler point must pass, this approach will be useful but the errors at the other points may be increased.

Sometimes it is possible that we can have more than one solution, in that case one can choose the suitable one regarding other properties, like the transmission quality, the space requirement, the sensitivity to the tolerance, the quick-return ratio etc.. If the designer gets a Grashofian double-rocker as a solution, then using the property of cognates one can have crank-rocker solutions for the same coupler curve.

[ iv ] Overall optimization approach for the function generation problem also gives good results, but it may require some trials on the initial values. Here, one can also visualize the nature of the function on the screen. If instead of a function, a set of coordinated inputs and outputs are given, then one can guess the type of the standard functions ( e.g. sinusoidal function ) by visualizing the curve ( $\theta_4$ vs. $\theta_2$ ).

[ v ] Regarding the two cusps problem, one can definitely obtain a crank-rocker mechanism with the proper selection of $x_{O2}$ and $x_{B1}$ ( section 4.1 ). We can also visualize the movement of the coupler point from one cusp to other on the screen itself.

[ vi ] We can draw all the coupler curves of 4R and 3R-1P mechanisms. The coupler curves can be drawn in segments for some specified crank rotation, by which one can predict the relative velocity at different positions.

[ vii ] The problems with kinematic constraints ( angular velocity, acceleration etc. ) can also be solved for the two cases discussed in chapter 5.

## 6.2 SUGGESTIONS FOR FUTURE WORK

[ i ] All the problems, which we have discussed for four-link mechanisms can arise in the four-link module of a six-link and other complex mechanisms. In that situation, by synthesizing the four-link module, one may be able to the synthesize the required mechanisms.

[ **ii** ] Besides the precision point and overall optimization approaches one can think about a *third approach* which includes the overall optimization and also takes care of some critical points. Some other constraints like the minimum transmission angle, tolerences etc. can also be included.

[ **iii** ] The weighted error function can also be used in the case of function generation problem and again one can consider the constraints which are mentioned in the path generation problem.

[ **iv** ] In the case of two cusps problem, since we can get more than one solution ( depending upon the assumptions $x_{O2}$ and $x_{B1}$ ), like dead-center problems of crank-rocker mechanisms one can maximize the minimum transmission angle to get a better crank-rocker mechanism.

[ **v** ] Like an atlas of coupler curves for crank-rocker mechanisms developed by Hrones and Nelson[11], one can think of storing all the coupler curves for 4R and 3R-1P mechanisms, which can be treated as a quick guide for the designers.

[ **vi** ] One can design a 4R mechanism if kinematic conditions ( Angular displacement, velocity and acceleration ) for input and floating links are prescribed. This can be done by writing Freudenstein's equation in terms of $\theta_2$ and $\theta_3$.

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# APPENDIX

## BASIC CLASSES OF C$^{++}$ USED IN THE SOFTWARE

In developing a large and complicated software, classes are necessarily used. In the present software, the basic classes used, are

**[i] Revolute** - It contains all the necessary characteristics of revolute pairs.

The main member functions of this class are

- EnterData ( ) ; This function asks for the link lengths through another class *LinkLength*.

- IfExist ( ) ; It checks for the existence of a chain

- FixLink ( ) ; After fixing a link chain becomes a mechanism.

- MaxMin ( ) ; Finds the maximum and minimum lengths.

- IsGrashof ( ) ; Checks for a grashofian mechanism.

- CalAngle ( ) ; Calculates the output angle for each input angle, which is used in animation of 4R mechanism.

- CalRange ( ) ; Calculates the range of the angular movement of any link of the mechanism.

- EnterOmega ( ) ; Asks for the input angular velocity.

- EnterAcc ( ) ; Asks for the input angular acceleration.

There is an inherited class *Prism* of the class *Revolute* for prismatic pairs.

**[ii] LinkLength** - It stores and returns the link length corresponding to a particular link.

**[iii] Range** - It stores and returns the range of linear or angular movement of any link of the mechanism.

[iv] **Animate** - Using this class animation of the mechanisms along with creating of menus and windows are done. The initial interaction with the user is made through a member function of this class. The main functions of this class are

- Draw_4R ( ) ; Animates 4R mechanisms.

- Draw_3R1P ( ) ; Animates 3R-1P mechanisms.

- Curve_4R ( ) ; Plots coupler curve for 4R mechanisms.

- Curve_3R1P ( ) ; Plots coupler curve for 3R-1P mechanisms.

- Cog4R ( ) ; Animates the cognates of crank-rocker and double-rocker mechanisms.

- Display( ) ; Calculates the minimum transmission angle and display it along with the link lengths and types of mechanisms.

- PlotTh4 ( ) ; It plots output angle vs input angles.

- PlotVelGraph ( ) ; Plots input and output angular velocities corresponding to each input angular displacement.

- GiveAcc ( ) ; It does acceleration analysis.

[v] **ScreenData** - This class interacts with the user and asks for the data's available through some member functions of class *synthesis1* to synthesis a mechanism for different types of problems.

[vi] **Angle** - It stores and returns the angular data's, which are used in synthesis of mechanisms.

[vii] **Point** - It stores and returns the linear data's or co-ordinates of points (required in path generation problems, cusp problems etc. ), which are used in synthesis of mechanisms.

[viii] **Synthesis1** - This class is mainly used to compute the unknown design parameters for all types of synthesis problems which we have already discussed in this work. the main functions of this class are

- AddPointData ( ) ; With the help of this, the required linear inputs for different types of problems are asked.

- AddAngleData ( ) ;  Same as above but this is for angular data 's.

- Deadsol ( )  ; Computes the unknown design parameters for dead centre problems.

- SolveEq1 ( ) ; For overall optimisation path generation problems

- SolveEq2 ( ) ;. For overall optimisation function generation problems.

- Cuspsol ( ) ; For two cusps problems.

- Kinsol ( ) ; For problems with kinematic constraints.

Thus in the case of synthesis of mechanisms, *ScreenData* interacts with the user and executes the member functions of *Synthesis1* ( the member functions of *Synthesis1* execute the functions of classes *Angle and Point* ) and transfer all the design parameters to class *Animate* for animation of the mechanism through EnterData ( ).